October 2, 2012 **Name** _____

The problems count as marked. The total number of points available is 169. Throughout this test, **show your work.**

1. (10 points) Find the exact value of $|5\sqrt{2} - 7| + |1 - 4\sqrt{2}| - |9\sqrt{2} - 11|$.

2. (10 points) The points $(2, k)$ and $(5, 5)$ belong to the line perpendicular to the line $6x - 2y = 7$. Find the value of k.

3. (35 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to\infty} \dfrac{3x^6 + x^4 - 6}{(11 - 3x^3)^2}$

   (b) $\lim_{x\to 2} \dfrac{x^2 - 4}{x^4 - 16}$

   (c) $\lim_{h\to 0} \dfrac{(1+h)^2 - 1}{h}$.

   (d) $\lim_{x\to 1} \dfrac{x^2 - 4x + 3}{x^2 + x - 2}$

   (e) $\lim_{x\to 2} \dfrac{\frac{1}{3x} - \frac{1}{6}}{\frac{1}{2x} - \frac{1}{4}}$

   (f) $\lim_{x\to -\infty} \dfrac{\sqrt{36x^2 - 3x}}{9x - 11}$

   (g) $\lim_{x\to 2} \dfrac{\sqrt{8x} - 4}{x - 2}$

4. (30 points) A topless box is constructed from a rectangular piece of cardboard that measures 16 inches by 12 inches. An x by x square is cut from each of the four corners, and the sides are then folded upwards to build the box.

   (a) Express the volume V as a function of x.

   (b) Use the physical constraints to find the domain of V.

   (c) Evaluate V at the $x = 1, x = 2$, and $x = 3$.

   (d) Find the derivative of V and use it to find the places where the tangent line is horizontal.

   (e) Find the critical points of V (ie, the places where the tangent line is horizontal) and pick out the one that belongs to the domain of V. Estimate this critical point to the nearest tenth of a unit. Estimate the value of V at that point.

5. (12 points) Find the domain of the function

$$g(x) = \sqrt{x(x+1)(x-1)(x-3)}.$$

Express your answer as a union of intervals. That is, use interval notation.

6. (12 points) Let $H(x) = (x^2-4)^2(x-3)^2$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2-4) \cdot 2x(x-3)^2 + (x^2-4)^2 \cdot 2(x-3).$$

Three of the zeros of $H'(x)$ are $x = \pm 2$ and $x = 3$. Find the other two.

7. (10 points) The demand curve for a new phone is given by $3p + 2x = 18$ where p is the price in hundreds of dollars and x is the number demanded in millions. The supply curve is given by $x - p^2 + 4p = 3$. Find the point of equilibrium.

8. (10 points) Suppose $p(x)$ is a polynomial of degree 5 and $q(x)$ is a polynomial of degree 6. What is the degree of the polynomial $H(x) = (x^2p(x) - 1)^2 - (q(x) + x^2)^2 + x^{13}$? Write a sentence about your reasoning.

9. (15 points) Let

$$f(x) = \begin{cases} |x-3| & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ x - 2 & \text{if } 2 < x \leq 4 \\ x^2 - 14 & \text{if } 4 < x \end{cases}$$

(a) What is $\lim_{x \to 2^-} f(x)$?

(b) What is $\lim_{x \to 2^+} f(x)$?

(c) What is $\lim_{x \to 2} f(x)$?

(d) What is $\lim_{x \to 4^-} f(x)$?

(e) What is $\lim_{x \to 4^+} f(x)$?

(f) What is $\lim_{x \to 4} f(x)$?

(g) What is $f(2)$?

(h) What is $f(4)$?

10. (25 points) Let $f(x) = \sqrt{2x+1}$. Notice that $f(4) = \sqrt{2 \cdot 4 + 1} = 3$.

    (a) Find the slope of the line joining the two points $(4, f(4))$ and $(5, f(5))$.

    (b) Let h be a positive number. What is the slope of the line passing through the points $(4, f(4)$ and $(4+h, f(4+h)$. Your answer depends on h, of course.

    (c) Compute $\lim_{h \to 0} \frac{f(4+h)-f(4)}{h}$ to get $f'(4)$.

    (d) Your answer to (c) is the slope of the line tangent to the graph of f at the point $(4, f(4))$. In other words, your answer is $f'(4)$. Write and equation for the tangent line.