February 6, 2013 **Name** ________________________________

The problems count as marked. The total number of points available is 149. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (30 points) Let L denote the line defined by the equation $2y + 3x = 12$.

   (a) Select a point P that belongs to the line. There are infinitely many correct answers to this.

   Solution: One point is $P = (2, 3)$ since $2 \cdot 3 + 3 \cdot 2 = 12$.

   (b) Find an equation for the line perpendicular to L that goes through the point you selected.

   Solution: The line must have slope $2/3$ since L has slope $-3/2$. Thus, using point-slope form, we have $y - 3 = (2/3)(x - 2)$ or $y = 2x/3 + 5/3$ in slope-intercept form.

   (c) Find the distance between your point P and the origin $(0, 0)$.

   Solution: $D = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{13}$.

   (d) Find the midpoint of the line segment with endpoints P and $(0, 0)$.

   Solution: The midpoint is $(\frac{0+2}{2}, \frac{0+3}{2}) = (1, 3/2)$.

2. (35 points) Evaluate each of the limits indicated below.

   (a) $\lim\limits_{x\to\infty} \dfrac{(2x^2-3)^2}{(x-1)^4}$
   Solution: The degree of both the numerator and the denominator is 4, so the limit is $4/1 = 4$.

   (b) $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-16}$
   Solution: Factor the denominator to get $\lim_{x\to 4} \frac{x-4}{(x-4)(x+4)} = \lim_{x\to 4} \frac{1}{(x+4)} = \frac{1}{8}$

   (c) $\lim\limits_{h\to 0} \dfrac{(4+h)^2-16}{h}$.
   Solution: Expand the numerator to get

$$\lim_{h\to 0} \frac{16+8h+h^2-16}{h} = \lim_{h\to 0} \frac{8h+h^2}{h} = \lim_{h\to 0} \frac{h(8+h)}{h}$$

   $= \lim\limits_{h\to 0}(8+h)$, and now the zero over zero problem has disappeared. So the limit is 8.

   (d) $\lim\limits_{x\to 1} \dfrac{x^2+3x-4}{x^2-3x+2}$
   Solution: Factor and eliminate the $x-1$ from numerator and denominator to get

$$\lim_{x\to 1} \frac{x+4}{x-2} = -5$$

   (e) $\lim\limits_{x\to 2} \dfrac{\frac{1}{4x}-\frac{1}{8}}{\frac{1}{2x}-\frac{1}{4}}$
   Solution: The limit of both the numerator and the denominator is 0, so we must either factor or do the fractional arithmetic. Factoring seems to work best. The limit becomes

$$\lim_{x\to 2} \frac{\frac{1}{2}[\frac{1}{2x}-\frac{1}{4}]}{[\frac{1}{2x}-\frac{1}{4}]} = \lim_{x\to 2} = \frac{1}{2}.$$

   (f) $\lim\limits_{x\to 5} \dfrac{\sqrt{3x+1}-4}{x-5}$
   Solution: Rationalize the numerator to get

$$\lim_{x\to 2} \frac{(\sqrt{3x+1}-4)(\sqrt{3x+1}+4)}{(x-5)(\sqrt{3x+1}+4)} = \frac{3(x-5)}{(x-5)(\sqrt{3x+1}+4)} \to \frac{3}{4+4} = \frac{3}{8}$$

3. (12 points) Consider the function g defined symbolically by

$$g(x) = \sqrt{(x+3)(2x-1)(x-5)}.$$

Note that $g(0) = \sqrt{15}$, so 0 belongs to the domain of g. Find the domain of the function. Express your answer as a union of intervals. That is, use interval notation.

Solution: The function is defined for those x for which $(x+3)(2x-1)(x-5) \geq 0$, so build the sign chart for $(x+3)(2x-1)(x-5)$ to see that it is at least zero on $[-3, 1/2] \cup [5, \infty)$.

4. (12 points) Let $H(x) = (x^2-1)^2(x+2)^2$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2-1) \cdot 2x(x+2)^2 + 2(x+2)(x^2-1)^2.$$

Please note that the derivative has already been found for you. There is no need to differentiate. Find all of the zeros of $H'(x)$. This is not a calculus problem. It's an algebra problem.

Solution: Factor out the common terms to get $H'(x) = 2(x^2-1)(x+2)[2x(x+2) + (x^2-1)]$. One factor is $2x^2 - 4x + x^2 - 1 = 3x^2 + 4x - 1$. Apply the quadratic formula to get $x = \frac{-4 \pm \sqrt{16+4(3)}}{6}$ which reduces to $x = \frac{-2 \pm \sqrt{7}}{3}$. The other 3 zeros of H' are $x = \pm 1$ and $x = -2$.

5. (10 points) The demand curve for a new phone is given by $3p + 2x = 12$ where p is the price in hundreds of dollars and x is the number demanded in millions. The supply curve is given by $x - p^2 + 4p = 4$. Find the point of equilibrium.

   Solution: Since $x = -3p/2 + 6$ and $x = p^2 - 4p + 4$, we can solve $-3p/2 + 6 = p^2 - 4p + 4$ for p: $p^2 - 4p + 3p/2 + 4 - 6 = 0$, so $2p^2 - 5p - 4 = 0$, which can be solved by the quadratic formula. We get $p = \frac{5}{4} \pm \frac{\sqrt{57}}{4}$. Taking the positive one of these ≈ 3.137) and solving for x yields $x \approx 4.294$.

6. (10 points) Find the exact value of $|2\sqrt{2} - 7| + |1 - 6\sqrt{2}| - |3\sqrt{2} - 11|$.

   Solution: Using the definition of absolute value, we have $7 - 2\sqrt{2} + 6\sqrt{2} - 1 - (11 - 3\sqrt{2}) = 7 - 1 - 11 - 2\sqrt{2} + 6\sqrt{2} + 3\sqrt{2} = -5 + 7\sqrt{2}$.

7. (15 points) Let

$$f(x) = \begin{cases} |x-2| & \text{if } x < -1 \\ 2x+5 & \text{if } -1 \leq x \leq 1 \\ x-2 & \text{if } 1 < x \leq 3 \\ x^2-8 & \text{if } 3 < x \end{cases}$$

(a) What is $f(-1)$?
Solution: $f(-1) = 3$.

(b) What is $f(1)$?
Solution: $f(1) = 7$.

(c) What is $f(3)$?
Solution: $f(3) = 1$.

(d) What is $\lim_{x\to -1^-} f(x)$?
Solution: $\lim_{x\to -1^-} f(x) = |-3| = 3$

(e) What is $\lim_{x\to 1^+} f(x)$?
Solution: $\lim_{x\to 1^+} f(x) = -1$

(f) What is $\lim_{x\to 3} f(x)$?
Solution: $\lim_{x\to 2} f(x) = 1$.

(g) List the x-values between -2 and 4 for which f is not continuous.
Solution: The function is discontinuous only at $x = 1$.

8. (25 points) Let $f(x) = \frac{1}{2x+1}$. Notice that $f(0) = 1$.

   (a) Find the slope of the line joining the two points $(0, f(0))$ and $(1, f(1))$.

   Solution: The slope is $\frac{f(1)-f(0)}{1-0} = \frac{\frac{1}{3}-1}{1} = -2/3$.

   (b) Let h be a positive number. What is the slope of the line passing through the points $(0, f(0)$ and $(0+h, f(0+h))$. Your answer depends on h, of course.

   Solution: $\frac{f(0+h)-f(0)}{h} = \frac{\frac{1}{2h+1}-1}{h}$.

   (c) Compute $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$ to get $f'(0)$.

   Solution: Since we get zero over zero, we recall that, in this case, we should rationalize the denominator.

$$\begin{aligned}\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} &= \lim_{h\to 0} \frac{\frac{1}{2h+1} - \frac{2h+1}{2h+1}}{h} \\ &= \lim_{h\to 0} \frac{-2h}{h(2h+1)} \\ &= -2\end{aligned}$$

   So, $f'(4) = 1/3$.

   (d) Your answer to (c) is the slope of the line tangent to the graph of f at the point $(0, f(0))$. In other words, your answer is $f'(0)$. Write an equation for that tangent line.

   Solution: So $f'(0) = -2$. The line is $y - 1 = -2(x-0)$, or $y = -2x + 1$.