October 19, 1999

Show your work on these problems.

1. Let $\frac{d}{dx}\{(3x+1)^3 + x^{-1} + x^{-3}\} = D$.

   Then $D = 3(3x+1)^2 \cdot 3 + (-1)x^{-2} + (-3)x^{-4} = 9(3x+1)^2 - x^{-2} - 3x^{-4}$

2. Let $\frac{d}{dx}(3x+1)(2x+5)(x^2-4) = D$.

   Then

$$\begin{aligned} D &= (x^2-4)\frac{d}{dx}[(3x+1)(2x+5)] + [(3x+1)(2x+5)]\frac{d}{dx}(x^2-4) \\ &= (x^2-4)[(3)(2x+5) + (3x+1)(2)] + [(3x+1)(2x+5)]2x \\ &= (x^2-4)[12x+17] + [6x^2+17x+5]2x \\ &= 12x^3 + 17x^2 - 48x - 68 + 12x^3 + 34x^2 + 10x \\ &= 24x^3 + 51x^2 - 38x - 68 \end{aligned}$$

3. Let $Q(x) = \left(\dfrac{x+2}{x+1}\right)^4$. Find $Q'(x)$

$$\begin{aligned} Q'(x) &= 4\left(\frac{x+2}{x+1}\right)^3 \cdot \frac{d}{dx}\left(\frac{x+2}{x+1}\right) \\ &= 4\left(\frac{x+2}{x+1}\right)^3 \cdot \frac{1\cdot(x+1) - 1\cdot(x+2)}{(x+1)^2} \\ &= 4\left(\frac{x+2}{x+1}\right)^3 \cdot \frac{-1}{(x+1)^2} \\ &= -4\frac{(x+2)^3}{(x+1)^5} \end{aligned}$$

4. Let $H(x) = \dfrac{f(x)}{(2x-3)^2}$, where $f(2) = 4$ and $f'(2) = 5$. What is $H'(2)$?

$$\begin{aligned} H'(x) &= \frac{f'(x)\cdot(2x-3)^2 - 2(2x-3)\cdot 2\cdot f(x)}{((2x-3)^2)^2} \\ H'(2) &= \left(\frac{f'(2)\cdot(2\cdot 2-3)^2 - 2(2\cdot 2-3)\cdot 2\cdot f(2)}{((2\cdot 2-3)^2)^2}\right) \\ &= \left(\frac{5\cdot 1 - 2(1)\cdot 2\cdot 4}{((1)^2)^2}\right) = -11 \end{aligned}$$

5. $\lim_{h \to 0} \frac{\sqrt{3x+3h} - \sqrt{3x}}{h}$ newline Rationalize the numerator by multiplying by $\frac{\sqrt{3x+3h}+\sqrt{3x}}{\sqrt{3x+3h}+\sqrt{3x}}$, resulting in

$$\lim_{h \to 0} \frac{3x + 3h - 3x}{h(\sqrt{3x+3h} + \sqrt{3x})}$$

which is the same as $\lim_{h \to 0} \frac{3h}{h(\sqrt{3x+3h} + \sqrt{3x})}$,
which in turn is $\lim_{h \to 0} \frac{3}{(\sqrt{3x+3h} + \sqrt{3x})}$. Now let $h \to 0$ to get $\frac{3}{2(\sqrt{3x})}$.