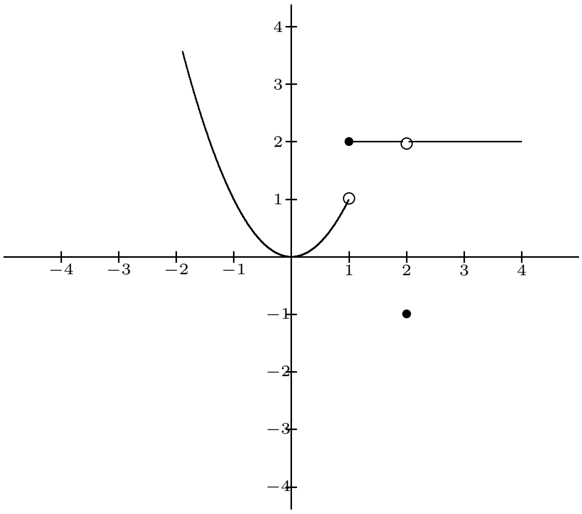
Math 1120 **Calculus** **Test 2**

July 20, 1999 Your name ____________________

On all the following questions, **show your work.**

1. (20 points) Answer the following questions about the function f whose graph is shown.



   (a) $\lim_{x \to 1} f(x) =$

   The limit does not exist because the left and right limits are different.

   (b) $\lim_{x \to 2^+} f(x) = \boxed{2}$

   (c) Estimate $f'(-1)$, and explain why your estimate is worthy.

   -2, the tangent line slopes downward and y changes about twice as fast as x.

   (d) Estimate $f'(0)$. Explain your answer.

   0, the tangent line seems to be horizontal.

2. (15 points)

   (a) State the hypothesis of the Intermediate Value Theorem (IVT).

   The theorem requires that the function f be continuous over an inteval $[a, b]$, and that M be a value of f between $f(a)$ and $f(b)$.

   (b) State the conclusion of the Intermediate Value Theorem.

   The conclusion is that there exists a number c such that $a < c < b$ and $f(c) = M$.

(c) Does the function $f(x) = \sqrt{x+3}$ satisfy the hypothesis of IVT over the interval $[1, 13]$. If so, find a whole number M between $f(1)$ and $f(13)$, and then find a number c in the interval $(1, 13)$ such that $f(c) = M$.

> The only integer between $f(1) = \sqrt{1+3} = 2$ and $f(13) = \sqrt{13+3} = 4$ is 3, so we need to solve the equation $f(c) = \sqrt{c+3} = 3$. Squaring both sides yields $c + 3 = 9$, and it follows that $c = 6$.

3. (20 points) Let $f(x) = 3x^2 + 1$

(a) Compute the derivative f' of f *using the definition of derivative.*

> Compute $\frac{f(x+h)-f(x)}{h}$, simplify and eliminate the h in both numerator and denominator to get $\lim_{h\to 0} \dfrac{3(2xh + h^2)}{h} = \lim_{h\to 0} 6x + h = 6x$.

(b) What is the slope of the line tangent to the graph of f at the point $(1, 4)$?

> $f'(1) = 6 \cdot 1 = 6.$

(c) Find an equation for the line tangent to the graph of f at the point $(1, 4)$

> $y - 4 = 6(x - 1)$ or $y = 6x - 2.$

4. (30 points) Compute the following derivatives.

(a) Let $f(x) = x^2 - 1/x$. Find $\frac{d}{dx} f(x)$.

> $f'(x) = 2x + \frac{1}{x^2}.$

(b) Let $g(x) = \sqrt{x^2 + 4}$. What is $g'(x)$?

> $g'(x) = \frac{1}{2}(x^2 + 4)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+4}}.$

(c) Find $\frac{d}{dx}(2x + 1)^4 \cdot (4x^2 - 1)$

> $\frac{d}{dx}(2x+1)^4 \cdot (4x^2-1) = (2x+1)^4(8x) + (4x^2-1) \cdot 4(2x+1)^3 \cdot 2$
> $= 8x(2x+1)^4 + 8(4x^2-1)(2x+1)^3.$

(d) Find $\frac{d}{dx}\frac{2x+1}{x^2+2}$

> $\frac{d}{dx}\frac{2x+1}{x^2+2} = \frac{2(x^2+2)-2x(2x+1)}{(x^2+2)^2} = \frac{-2x^2-2x+4}{(x^2+2)^2}.$

(e) Find $\frac{d}{dt}(t^{-1} + t^{-2})^3$.

> $\frac{d}{dt}(t^{-1} + t^{-2})^3 = 3(t^{-1} + t^{-2})^2(-t^{-2} - 2t^{-3}).$

5. (20 points) Let $C(x) = 8000 + 200x - 0.1x^2,$ for $0 \le x \le 400$ be the cost in dollars of producing x air conditioners.

   (a) Find the cost of producing the 301st air conditioner. $\boxed{C(301) - C(300) = 139.9}$

   (b) Find the average cost function $\overline{C}(x)$. $\boxed{\overline{C}(x) = \frac{8000+200x-0.1x^2}{x}.}$

   (c) Find the rate of change of the cost with respect to x when $x = 300$. $\boxed{C'(x) = 200 - 0.2x, \text{ so } C'(300) = 200 - 0.2(300) = 200 - 60 = 140.}$