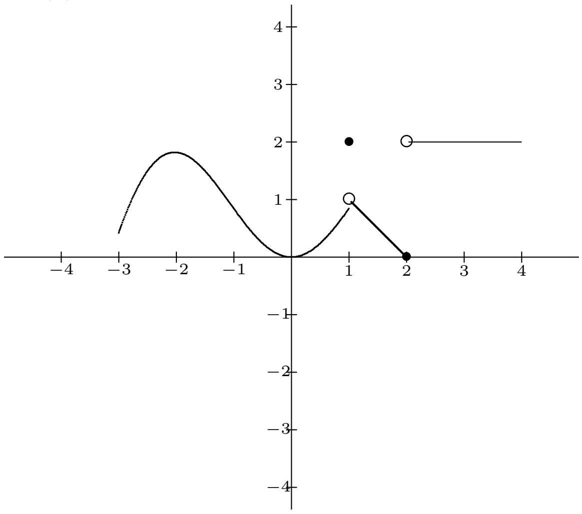
February 23, 2001 **Name** ____________________________

In the first 3 problems, each part counts 6 points (total 42 points) and the final 4 problems count as marked. The total number of points available is 112.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Questions (a) through (e) refer to the graph of the function f given below.



(a) $\lim_{x\to 1} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

Solution: Use the blotter test by covering up the left part and then the right part to determine the one-sided limits, both of which are 1. Therefore, $\lim_{x\to 1} f(x) = 1$.

(b) $\lim_{x\to 2^-} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

Solution: Again use the blotter test by covering up the right part to determine the one-sided limits, both of which are 1. Therefore, $\lim_{x\to 2^-} f(x) = 0$

(c) A good estimate of $f'(0)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: The tangent line seems to be horizontal, therefore the best estimate of its slope is 0.

(d) A good estimate of $f'(-1)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: The tangent line has a negative slope, therefore the best estimate of its slope is -1.

(e) A good estimate of $f'(2)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: There is no tangent line, so there is no good estimate of $f'(2)$.

2. The line tangent to the graph of a function f at the point $(2, 3)$ on the graph also goes through the point $(-2, 7)$. What is $f'(2)$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

Solution: The slope of the tangent line is $\frac{2-(-2)}{3-7} = -1$, so $f'(2) = -1$.

3. What is the slope of the tangent line to the graph of $f(x) = 2x^{-2}$ at the point (2,1/2)?

(A) $-1/2$ **(B)** $-1/4$ **(C)** $-1/8$ **(D)** $-1/16$ **(E)** $-1/512$

Solution: $f'(x) = -4x^{-3}$, which at $x = 2$ has the value $-1/2$.

On all the following questions, **show your work.**

4. (15 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \le c \le b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{2x-1}$ and let $[a, b] = [1, 5]$. Finally, suppose $M = 2$. Find the number c whose existence is guaranteed by IVT.

Solution: Solve the equation $\sqrt{2x-1} = 2$ by squaring both sides. You get $x = 5/2$.

5. (15 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing x playing records is given by $C(x) = 2000 + 3x - 0.01x^2$ for x in the range 0 to 6000.

   (a) Find the marginal cost function $C'(x)$.
   Solution: $C'(x) = 3 - 0.02x$

   (b) Find the average cost function $\overline{C}(x)$.
   Solution: $\overline{C}(x) = \frac{2000+3x-0.01x^2}{x} = \frac{2000}{x} + 3 - 0.01x$.

   (c) Find the marginal average cost function $\overline{C}'(x)$.
   Solution: $\overline{C}'(x) = -2000x^{-2} - 0.01$.

   (d) Interpret your results in (c). Is the average cost growing or falling as the company produces more units?
   Solution: The function $\overline{C}'(x) = -2000x^{-2} - 0.01$ is negative throughout its domain. This means that the average cost decreases the more records are produced.

6. (15 points) Let $f(x) = 4/x$.

   (a) Construct $\frac{f(3+h)-f(3)}{h}$
   Solution: Note that $\frac{f(3+h)-f(3)}{h} = \frac{\frac{4}{3+h} - \frac{4}{3}}{h} = \frac{\frac{4\cdot 3}{(3+h)\cdot 3} - \frac{4(3+h)}{3(3+h)}}{h} = \frac{12-4(3+h)}{3(3+h)h}$.

   (b) Simplify and take the limit of the expression in (a) as h approaches 0 to find $f'(3)$.

Solution: continued from above: $= \frac{12-12-4h}{3(3+h)h} = \frac{-4h}{3(3+h)h} = \frac{-4}{3(3+h)}$. Therefore, $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \dfrac{-4}{3(3+h)} = -\dfrac{4}{9}$.

(c) Use the information found in (b) to find an equation for the line tangent to the graph of f at the point $(3, 4/3)$.

Solution: Use the point-slope formula to get $y - 4/3 = -4/9(x-3)$ which in slope-intercept form is $y = -\frac{4}{9}x + \frac{8}{3}$.

7. (25 points) Compute the following derivatives.

(a) Let $f(x) = x^3 + x^{-\frac{1}{2}}$. Find $\frac{d}{dx}f(x)$.

Solution: $\frac{d}{dx}f(x) = 3x^2 - \frac{1}{2}x^{-\frac{3}{2}}$.

(b) Let $g(x) = \sqrt{x^2+4}$. What is $g'(x)$?

Solution: $g'(x) = \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x$.

(c) Find $\frac{d}{dx}((3x+1)^2 \cdot (4x^4-1))$

Solution: $\frac{d}{dx}((3x+1)^2 \cdot (4x^4-1)) = \frac{d}{dx}((3x+1)^2) \cdot (4x^4-1) + (3x+1)^2 \frac{d}{dx}(4x^4-1) = 2(3x+1)(4x^4-1) + (3x+1)^2 \cdot 16x^3$.

(d) Find $\frac{d}{dx}\frac{2x^2+1}{x+2}$

Solution: Use the quotient rule to get $\frac{(4x+1)(x+2)-(2x^2+1)}{(x+2)^2} = \frac{2x^2+9x}{(x+2)^2}$.

(e) Find $\frac{d}{dt}(t^2+1/t)^2$.

Solution: $\frac{d}{dt}(t^2+1/t)^2 = 2(t^2+t^{-1})(2t-t^{-2}) = 2(2t^3+1-t^{-3})$.