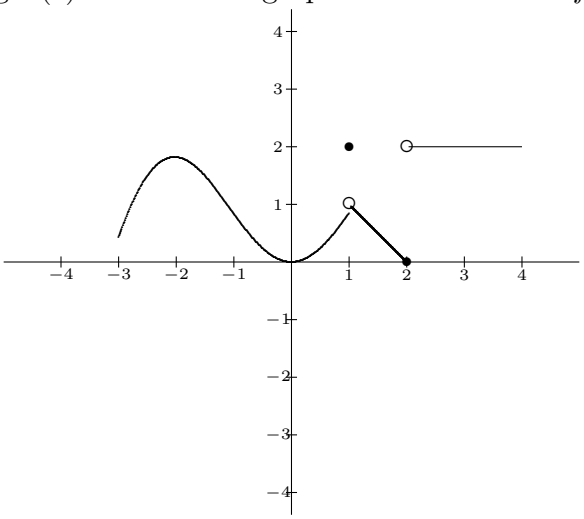
October 18, 2001 Your name _____

The multiple choice problems count 4 points each. In the multiple choice section, circle the correct choice (or choices). You must show your work on the other problems 5 through 10. The total number of points available is 131.

1. Questions (a) through (e) refer to the graph of the function f given below.



(a) $\lim_{x\to 3} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

(b) $\lim_{x\to 2^-} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

(c) A good estimate of $f'(-2)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

(d) A good estimate of $f'(-1)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

(e) A good estimate of $f'(3)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

2. The line tangent to the graph of a function f at the point $(2, 3)$ on the graph also goes through the point $(-1, 9)$. What is $f'(2)$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

3. What is the slope of the tangent line to the graph of $f(x) = (3x)^{-1}$ at the point (1,1/3)?

   (A) $-2/9$ **(B)** $-1/3$ **(C)** $-2/27$ **(D)** $-2/81$ **(E)** 0

4. True-false questions. These count 2 points each.

   (a) True or false. If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.

   (b) True or false. If $f(a) < 0, f(b) > 0$, and $f'(x) > 0$ for each x in (a, b), then there is one and only one number c in (a, b) such that $f(c) = 0$.

   (c) True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of f.

   (d) True or false. If $f'(c) = 0$, then f has a relative maximum or a relative minimum at $x = c$.

   (e) True or false. If f has a relative maximum or a relative minimum at $x = c$, then $f'(c) = 0$.

   (f) True or false. If $f'(c) = 0$ and $f''(c) < 0$, then f has a relative maximum at $x = c$.

   (g) True or false. If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.

   (h) True or false. If f and g are differentiable, then $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)}{g'(x)}$.

   (i) True or false. If f and g are differentiable and $h(x) = f \circ g$, then $h'(x) = f[g(x)]g'(x)$.

   (j) If f and g are differentiable and a and b are constants, then $\frac{d}{dx}[af(x) + bg(x)] = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$.

On all the following questions, **show your work.**

5. (10 points) Let $f(x) = 1/(2x)$.

   (a) Construct $\frac{f(2+h)-f(2)}{h}$

   (b) Simplify and take the limit of the expression in (a) as h approaches 0 to find $f'(2)$.

   (c) Use the information found in (b) to find an equation for the line tangent to the graph of f at the point $(2, 1/4)$.

6. (8 points) Suppose $f(x)$ is a function such that $f(2) = 1$ and $f'(x) = 3x + 4$ for all real numbers x. Let L denote the line that is tangent to the graph of $f(x)$ at the point $(2, 1)$. What is the slope of L? What is the y-intercept of L? What is the x-intercept of L?

7. (8 points) Find an equation of the tangent line to the graph of $f(x) = \sqrt{2x - 5}$ at the point (3,1).

8. (15 points)

   (a) State the hypothesis of the Intermediate Value Theorem (IVT).

   (b) State the conclusion of the Intermediate Value Theorem.

   (c) Does the function $f(x) = \sqrt{x+4}$ satisfy the hypothesis of IVT over the interval $[0, 12]$. If so, find a whole number M between $f(0)$ and $f(12)$, and then find a number c in the interval $(0, 12)$ such that $f(c) = M$.

9. (12 points) Suppose the functions f and g and their derivatives are given by the table of values shown. Complete the table by calculating the values of the derivatives of both $f \circ g(x)$ and $g \circ f(x)$ for each of the values of x in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$\frac{df \circ g(x)}{dx}$	$\frac{dg \circ f(x)}{dx}$
0	2	3	1	3		
1	3	4	5	2		
2	2	1	1	4		
3	5	3	4	1		
4	4	1	3	2		
5	2	0	0	4		

10. (30 points) Compute the following derivatives.

(a) Let $f(x) = x^2 - (1/x)$. Find $\frac{d}{dx}f(x)$.

(b) Let $g(x) = \sqrt{3x^3 + 4}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((2x+1)^3 \cdot (3x^2 - 1))$

(d) Find $\frac{d}{dx}\frac{2x+1}{x^2+2}$

(e) Find $\frac{d}{dt}(t^{-3} + t^{-2})^3$.