November 4, 2015 **Name** ______________________________

The problems count as marked. The total number of points available is 147. Throughout this test, **show your work.** Use of calculator to circumvent ideas discussed in class will generally result in no credit.

1. (20 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. No need to simplify. You must show your work.

   (a) Let $F(x) = (x^2 - 3x + 1)(x^3 - 2x + 5)$

   (b) $G(x) = \dfrac{2x^4 - 3x + 1}{x^2 - x + 3}$

   (c) $K(x) = (x^2 - 3)^{17}$

   (d) $H(x) = \sqrt{(3x + 1)^4 - 7}$.

2. (12 points) Mike thinks of a positive number. He adds the square of his number and the reciprocal of his number. What is the smallest possible sum he could obtain? A calculator solution will get no credit. Estimate the answer to the nearest 0.01.

3. (10 points) The line tangent to the graph of $g(x)$ at the point $(4, 7)$ has a y-intercept of 9. What is $g'(4)$?

4. (10 points) Find a point on the graph of $h(x) = x^3 - 6x^2 + 9x$ where the tangent line is horizontal. There are two such points on the graph of $h(x)$.

5. (10 points) Suppose f and g are functions for which both $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$. Which of the following could be true? Circle all the options that could be true.

   (A) $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = 0$ **(B)** $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist **(C)** $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = -1$

   (D) $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = 7$ **(E)** $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = 1$

6. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = (f(x) + g(x))^2$. Compute $L(2)$ and $L'(2)$.

(b) Let $U(x) = f \circ f \circ f(x)$. Compute $U(1)$ and $U'(1)$.

(c) Let $K(x) = g(x) + f(x^2)$. Compute $K(2)$ and $K'(2)$

(d) Let $Z(x) = 1/g(2x)$. Compute $Z(3)$ and $Z'(3)$.

(e) Let $Q(x) = g(3x) + f(2x)$. Compute $Q(2)$ and $Q'(2)$.

7. (20 points) Consider the fourth degree polynomial $p(x) = (x^2 - 4)^2$.

   (a) Find the intervals over which $p(x)$ is increasing.

   (b) Find the intervals over which $p(x)$ is concave upwards.

8. (30 points) Consider the function

$$r(x) = \frac{(x^2-4)(6x)}{(3x-6)(x+1)(x-3)}.$$

Use the Test Interval Technique to find the sign chart of $r(x)$. Find the zeros and the horizontal and vertical asymptotes, and sketch the graph of r. Your graph must be consistent with the information you find in the sign chart.