November 2, 2016 **Name** ______________________________

The problems count as marked. The total number of points available is 166. Throughout this test, **show your work.** This is an amalgamation of the tests from sections 3 and 10.

1. (30 points) Let $p(x) = 2x^3 + 3x^2 - 12x$

   (a) Find an equation for the line tangent to $p(x)$ at the point $(1, p(1))$.

   (b) Find an interval over which $p(x)$ is decreasing.

   (c) Find an interval over which $p(x)$ is concave upwards.

2. (12 points) Let $f(x) = \sqrt{x^2 - x + 3}$.

   (a) Compute $f'(x)$

   (b) What is $f'(3)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

3. (16 points) Polynomials f, g, and h have degrees 4, 5, and 6 respectively. For each polynomial below, find the degree. The symbol $\circ$ means composition.

   (a) $f \cdot g + g \cdot h + h \cdot f$ ________________

   (b) $f \circ (g + h)$ ________________

   (c) $f \circ (g \circ h)$ ________________

   (d) $f \circ (g \cdot h)$ ________________

4. (30 points) Let $H(x) = (x^2 - 9)^2(3x+1)^3$.

   (a) Use the chain and product rules to find $H'(x)$.

   (b) Find the critical points of H.

   (c) Build the sign chart for $H'(x)$

   (d) Classify the critical points of H as max, min, or imposters.

   (e) Find the intervals over which H is increasing.

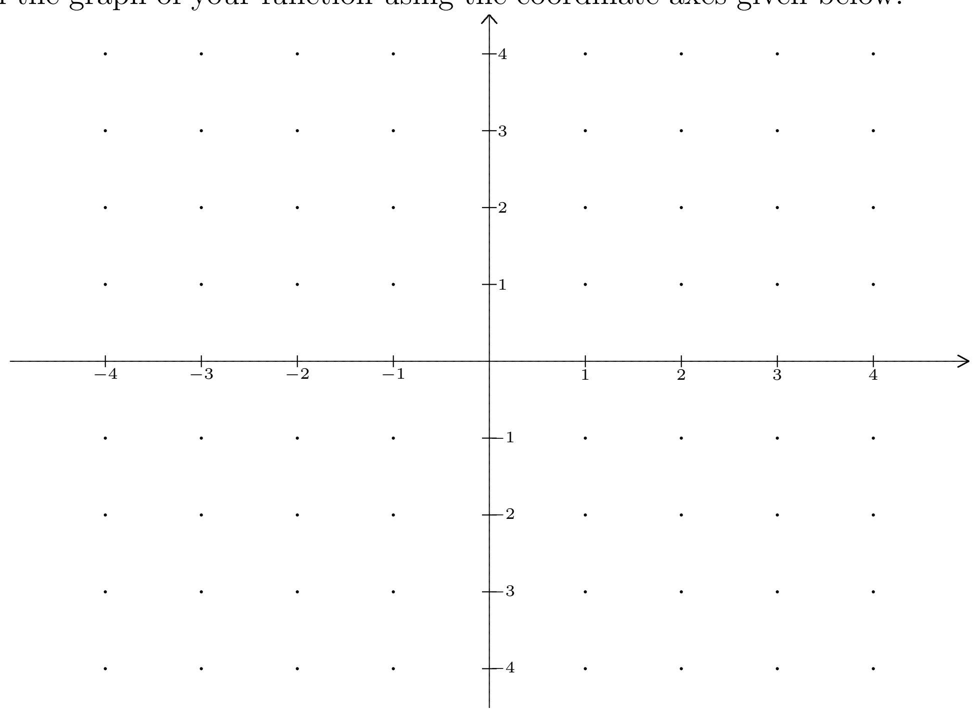
5. (30 points) Build a rational function $r(x)$ with the following properties:

   - r has vertical asymptotes at $x = -1$ and $x = 1$.
   - r has zeros at $x = -2$ and $x = 0$.
   - r satisfies $\lim_{x\to\infty} r(x) = 2$.

   (a) Find a symbolic representation for r.

   (b) Build the sign chart for your function.

   (c) Sketch the graph of your function using the coordinate axes given below.

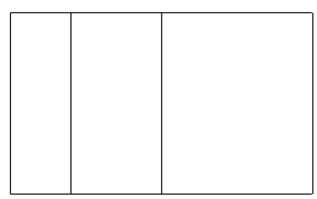


   (d) Find $r'(x)$ for your r.

   (e) Does r have any critical points? If so, find them.

6. (12 points) Mike thinks of a positive number. He adds the square and the reciprocal of his number. What is the least possible sum Mike could get?

7. (12 points) A farmer has 20000 feet of fencing to build a rectangular pasture. But he must separate the goats, horses and cows into different parts of the pasture using two vertical straight sections of fence as shown.



What is the area of the largest pasture the farmer can build?

8. (24 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	3	4	2	3
3	1	2	5	3
4	3	5	2	5
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = f(2x) \cdot g(x)$. What is $L(2)$? Compute $L'(2)$. Find an equation for the line tangent to L at the point $(2, L(2))$.

(b) Let $U(x) = \sqrt{g(x)}$. Compute $U(5)$ and $U'(5)$. Is the function U increasing or decreasing at $x = 5$?

(c) Let $K(x) = (g(x) + f(x))^2$. Compute $K(1)$ and $K'(1)$