

November 1, 2017

Name \_\_\_\_\_

The total number of points available is 152. Throughout this test, **show your work**.

1. (10 points) Let  $f(x) = x^3 - 2x - 3$ .

(a) Compute  $f'(x)$

(b) What is  $f'(2)$ ?

(c) Use the information in (b) to find an equation for the line tangent to the graph of  $f$  at the point  $(2, f(2))$ .

2. (12 points) Consider the function  $f$  defined by:

$$f(x) = \begin{cases} x + x^3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x^{1/2} & \text{if } x > 1 \end{cases}$$

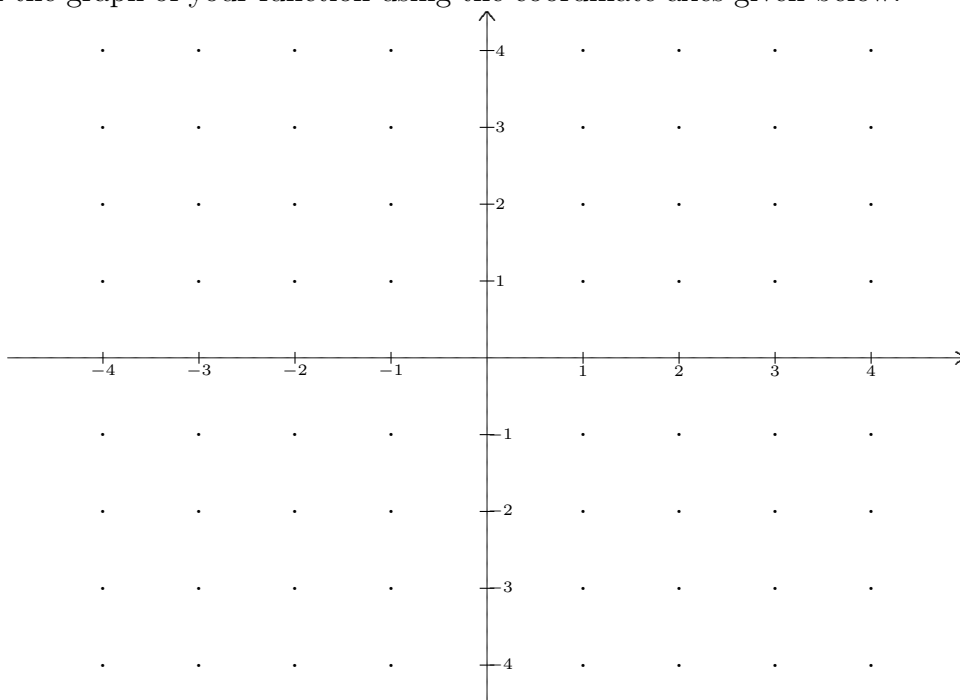
(a) Is  $f$  continuous at  $x = 1$ ?

(b) What is the slope of the line tangent to the graph of  $f$  at the point  $(4, 4)$ ?

(c) Find  $f'(-3)$

3. (25 points) If a stone is shot vertically upward from the roof of 212 foot building with a velocity of 320 ft/sec, its height after  $t$  seconds is  $s(t) = 212 + 320t - 16t^2$ .
- (a) What is the height the stone at time  $t = 0$ ?
  - (b) What is the height the stone at time  $t = 2$ ?
  - (c) What is the average velocity of the stone during the third second?
  - (d) What is the average velocity of the stone during time interval  $[2, 2.1]$ ?
  - (e) What is the average velocity of the stone during time interval  $[2, 2.01]$ ?
  - (f) What is  $s'(2)$ ?
  - (g) What is the velocity of the stone at the time it reaches its maximum height?
  - (h) At what time is the velocity zero?
  - (i) What is the maximum height the stone reaches?
  - (j) What is the velocity of the stone when it hits the ground (height 0)?

4. (20 points) Let  $f(x) = (x^2 - 9)^{2/3}$ . Note: some tests had the function  $f(x) = (x^2 - 9)^{1/3}$  or similar variations. These two types of functions yield quite different answers.
- (a) What is the domain of  $f$ ?
- (b) Find all the critical points of  $f$
- (c) Identify each critical point of  $f$  as relative minimum, a relative maximum, or an imposter.
- (d) Build the sign chart for your function.
- (e) Sketch the graph of your function using the coordinate axes given below.



5. (30 points) Consider the table of values given for the functions  $f, f', g$ , and  $g'$ :

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

- (a) Let  $L(x) = f(x + 1) + g(x - 1)$ . Compute  $L(2)$  and  $L'(2)$ .

- (b) Let  $U(x) = g \circ f(x)$ . Compute  $U(1)$  and  $U'(1)$ .

- (c) Let  $K(x) = g(x) \cdot f(x^2)$ . Compute  $K(2)$  and  $K'(2)$ .

- (d) Again,  $L(x) = g(x + 2) \div f(2x - 1)$ . Compute  $L(2)$  and  $L'(2)$ .

- (e) Let  $Z(x) = g(x^2 + f(x))$ . Compute  $Z(1)$  and  $Z'(1)$ .

6. (15 points) Two positive numbers  $x$  and  $y$  are related by  $2x + 3y = 16$ . What is the largest possible product  $xy$  could be, and what pair  $(x, y)$  achieves that product? Note that if  $y = 2$ , then  $x = 5$  and the product  $xy = 10$ . If  $y = 4$ , then  $x = 2$  and the product is 8. Trying various combinations of values is not worth any credit.

7. (10 points) The line tangent to the graph of a function  $f$  at the point  $(2, 9)$  on the graph also goes through the point  $(0, 7)$ . What is  $f'(2)$ ?

8. (30 points) Let  $H(x) = (x^2 - 9)^2(3x + 1)^3$ .

(a) Use the chain and product rules to find  $H'(x)$ .

(b) Find the critical points of  $H$ .

(c) Build the sign chart for  $H'(x)$

(d) Classify the critical points of  $H$  as max, min, or imposters.

(e) Find the intervals over which  $H$  is increasing.

9. (20 points) Let  $f(x) = x^3 + x - 3$ . Prove that  $f$  has exactly one zero as follows.

(a) Use the Intermediate Value Theorem to show that  $f$  has at least one zero.

(b) Prove that  $f$  is an increasing function on its domain. Conclude that  $f$  cannot have more than one zero.