October 7, 1999 **Name** ____________________________

The first five problems counts 6 points each and the others count as marked.
Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Consider the function f defined by:

$$f(x) = \begin{cases} |x+2| & \text{if } x \le 0 \\ 5 - x^2 & \text{if } x > 0 \end{cases}$$

   Find the three solutions to $f(x) = 1$ and compute their sum.

   (A) -4 **(B)** $\boxed{-2}$ **(C)** 0 **(D)** 2 **(E)** 6

2. Let $f(x) = 1/x$. What is the vaule of $\dfrac{f(x+2) - f(x)}{2}$?

   (A) $\boxed{-\dfrac{1}{x(x+2)}}$ **(B)** $\dfrac{1}{x(x+2)}$ **(C)** $\dfrac{x}{x+2}$ **(D)** $-\dfrac{x}{x+2}$ **(E)** $x+2$

3. Let $f(x) = \sqrt{2x}$. What is the value of $f(x+1) - f(x)$ in terms of x?

   (A) $\boxed{\dfrac{2}{\sqrt{2x+2} + \sqrt{2x}}}$ **(B)** $\dfrac{2}{\sqrt{2x+1} + \sqrt{2x}}$ **(C)** $\dfrac{1}{\sqrt{2x+1}}$

   (D) $\sqrt{2x+2}$ **(E)** $\sqrt{2x+2} - x$

4. Suppose the point $(2, 5)$ belongs to the graph of a function g and $g'(2) = 4$. What is the y-intercept of the line tangent to the graph of g at the point $(2, 5)$?

   (A) -8 **(B)** $\boxed{-3}$ **(C)** 3 **(D)** 8 **(E)** 13

5. The line tangent to the graph of a function h at the point $(3, 7)$ has a y-intercept of 10. What is $h'(3)$?

   (A) -7 **(B)** -4 **(C)** $\boxed{-1}$ **(D)** 1 **(E)** $17/3$

6. (20 points) Let

$$f(x) = \begin{cases} 2x - 3 & \text{if } x \leq 4 \\ 6 - x & \text{if } x > 4 \end{cases}$$

and let $g(x) = 2x$.

(a) Compute each of the following

i. $f \circ g(1)$ $\boxed{f \circ g(1) = f(2) = 1}$

ii. $f \circ g(2)$ $\boxed{f \circ g(2) = f(4) = 5}$

iii. $f \circ g(3)$ $\boxed{f \circ g(3) = f(6) = 0}$

iv. $f \circ g(3.5)$ $\boxed{f \circ g(3.5) = f(7) = -1}$

(b) Find a symbolic representation of the composition $f \circ g(x)$, and simplify the representation.

$$f \circ g(x) = \begin{cases} 2(2x) - 3 & \text{if } 2x \leq 4 \\ 6 - 2x & \text{if } 2x > 4 \end{cases}$$

which is the same as

$$f \circ g(x) = \begin{cases} 4x - 3 & \text{if } x \leq 2 \\ 6 - 2x & \text{if } x > 2 \end{cases}$$

7. (25 points) Compute the limits requested.

(a) $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

Rationalize the numerator to get $\frac{(2+h)-2}{h(\sqrt{2+h}+\sqrt{2})} = \frac{h}{h(\sqrt{2+h}+\sqrt{2})}$ $= \frac{1}{(\sqrt{2+h}+\sqrt{2})}$, so the limit is just the value of the last expression at $h = 0$, which is $\frac{1}{2\sqrt{2}}$.

(b) $\lim_{x\to 3} \frac{x-3}{x^3-27}$

Factor the denominator to get $\frac{x-3}{(x-3)(x^2+3x+9)}$. The $(x-3)$ factors can be removed to give $\lim_{x\to 3} \frac{1}{x^2+3x+9} = 1/27$.

(c) $\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

Find a common denominator and simplify to get $\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$ $= \lim_{h\to 0} \frac{3-(3+h)}{h\cdot 3(3+h)}$ which is just $\lim_{h\to 0} \frac{-1}{3(3+h)} = -1/9$.

(d) $\lim_{x\to\infty} \frac{2x^3-2x^2+7}{4x^3-10x^2+x-27}$

Since the degrees of the numerator and denominator are the same, take the ratio of the coefficients of these highest power terms, $2x^3$ and $4x^3$ to get the result $\lim = 1/2$.

(e) $\lim_{x\to-\infty} \frac{|x|-3}{3x+5}$

Divide both numerator and denominator by x to get $\lim_{x\to-\infty} \frac{|x|-3}{3x+5} =$ $\lim_{x\to-\infty} \frac{|x|/x-3/x}{3x/x+5/x}$. This reduces to $\lim_{x\to-\infty} \frac{|x|/x}{3}$ whose value, for negative values of x, is $-1/3$.

8. (25 points) Find the following derivatives.

(a) $\frac{d}{dx}\sqrt{2x^3-5x+7}$

$\frac{d}{dx}\sqrt{2x^3-5x+7} = \frac{1}{2}(2x^3-5x+7)^{-\frac{1}{2}} \cdot (6x^2-5)$

(b) $\frac{d}{dx}(2x-1)\cdot(3x^2+4x)$

$$\begin{aligned}\frac{d}{dx}(2x-1)\cdot(3x^2+4x) &= (2x-1)(6x+4)+(3x^2+4x)\cdot 2\\ &= 12x^2+8x-6x-4+6x^2+8x\\ &= 18x^2+10x-4\end{aligned}$$

Alternatively, multiply the two factors and differentiate the result.

(c) $\frac{d}{dx}\frac{2x^2-1}{3x+2}$

$$\begin{aligned}\frac{d}{dx}\frac{2x^2-1}{3x+2} &= \frac{4x(3x+2)-3(2x^2-1)}{(3x+2)^2}\\ &= \frac{12x^2+8x-6x^2+3}{(3x+2)^2}\\ &= \frac{6x^2+8x+3}{(3x+2)^2}\end{aligned}$$

(d) $\frac{d}{dx}\sqrt{x^2-2x+1}$

$\frac{d}{dx}\sqrt{x^2-2x+1} = \frac{d}{dx}\sqrt{(x-1)^2} = \frac{d}{dx}|x-1| = \frac{|x-1|}{x-1}$

(e) $\frac{d}{dx}(x^3+3x^2+3x+1)^{1/3}$

$\frac{d}{dx}(x^3+3x^2+3x+1)^{1/3} = \frac{d}{dx}((x+1)^3)^{1/3} = \frac{d}{dx}(x+1) = 1$

9. (20 points) Let $f(x) = \frac{1}{x} + x$.

   (a) Compute $f(3.1)$ $\boxed{f(3.1) = \frac{1}{3.1} + 3.1 \approx 3.42}$

   (b) Compute $f(3+h)$ $\boxed{f(3+h) = \frac{1}{3+h} + 3 + h \approx 3.42}$

   (c) Compute $\frac{f(3+h)-f(3)}{h}$ and simplify, assuming $h \neq 0$.

$$\begin{aligned}
\frac{f(3+h)-f(3)}{h} &= \frac{\frac{1}{3+h} + 3 + h - (\frac{1}{3} + 3)}{h} \\
&= \frac{\frac{1}{3+h} - \frac{1}{3} + h}{h} \\
&= \frac{3-(3+h)}{h \cdot 3 \cdot (3+h)} + 1 \\
&= -\frac{h}{3h(3+h)} + 1 \\
&= -\frac{1}{3(3+h)} + 1
\end{aligned}$$

   (d) Take the limit of the expression in (c) as h approaches 0 to find $f'(3)$.

   The limit as h approaches 0 is $-\frac{1}{9} + 1 = \frac{8}{9}$.

   (e) What is the slope of the line tangent to f at the point $(3, 3\frac{1}{3})$.

   Its the number we just calculated, $\frac{8}{9}$.

   (f) Find an equation for the line tangent to the graph of f at the point $(3, 3\frac{1}{3})$.

   The equation is $y - 3\frac{1}{3} = \frac{8}{9}(x-3)$, which simplifies to $y = \frac{8}{9}x + \frac{2}{3}$.