

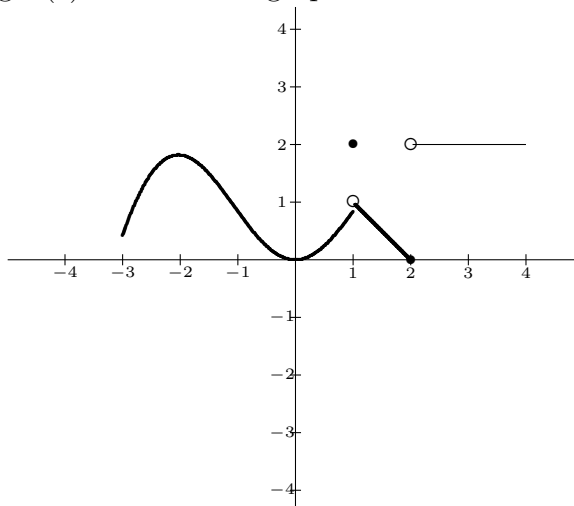
March 7, 2003

Name _____

The first 6 problems count 5 points each and the final 4 count as marked. The total number of points available is 128.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Questions (a) through (e) refer to the graph of the function f given below.



- (a) $\lim_{x \rightarrow 1} f(x) =$
 (A) 0 (B) 1 (C) 2 (D) 4 (E) does not exist
- (b) $\lim_{x \rightarrow 2^+} f(x) =$
 (A) 0 (B) 1 (C) 2 (D) 4 (E) does not exist
- (c) A good estimate of $f'(-1)$ is
 (A) -1 (B) 0 (C) 1 (D) 2 (E) there is no good estimate
- (d) A good estimate of $f'(-2)$ is
 (A) -1 (B) 0 (C) 1 (D) 2 (E) there is no good estimate
- (e) A good estimate of $f'(3)$ is
 (A) -1 (B) 0 (C) 1 (D) 2 (E) there is no good estimate
2. The line tangent to the graph of a function f at the point $(2, -3)$ on the graph also goes through the point $(-1, 6)$. What is $f'(2)$?
 (A) -3 (B) -1 (C) 0 (D) 1 (E) 3

3. True-false questions. These count 2 points each.
- (a) True or false. If f and g are differentiable and a and b are constants, then $\frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x)$.
- (b) True or false. If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.
- (c) True or false. If $f(a) < 0$, $f(b) > 0$, and $f(x)$ is continuous for each x in $[a, b]$, then there is at least one number c in (a, b) such that $f(c) = 0$.
- (d) True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of f .
- (e) True or false. If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.
- (f) True or false. If f and g are differentiable, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)}{g'(x)}$.
- (g) True or false. If f and g are differentiable and $h(x) = f \circ g$, then $h'(x) = f(g(x))g'(x)$.
4. (40 points) Suppose the functions f and g have derivatives at all their domain points and their values at certain points are given in the table. The next four problems refer to these functions f and g . Recall that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. In each case, a function $H(x)$ is given in terms of $f(x)$ and $g(x)$. You are asked to find $H'(x)$ at the value of x provided.

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 0 | 2 | 1 | 5 | 4 |
| 1 | 7 | 3 | 6 | 2 |
| 2 | 5 | 4 | 1 | 7 |
| 3 | 1 | 2 | 6 | 8 |
| 4 | 3 | 3 | 2 | 5 |
| 5 | 6 | 4 | 1 | 4 |
| 6 | 0 | 5 | 4 | 6 |
| 7 | 4 | 1 | 5 | 1 |

- (a) The function H is defined by $H(x) = f(x^2)$. Find $H'(1)$.
(A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 44
- (b) The function J is defined by $J(x) = f(g(f(x)))$. Use the chain rule to find $J'(2)$.
(A) 6 **(B)** 9 **(C)** 12 **(D)** 21 **(E)** 48

- (c) The function K is defined by $K(x) = g(x)/x^2$. Use the quotient and chain rules to find $K'(3)$.
- (A) $-1/9$ (B) $1/3$ (C) $2/3$ (D) $4/9$ (E) $7/9$
- (d) The function L is defined by $L(x) = (x + f(x))^{10}$. Use the chain and power rules to find $L'(0)$.
- (A) 0 (B) $10 \cdot 2^9$ (C) $5 \cdot 2^9$ (D) $10 \cdot 2^{10}$ (E) 2^{11}
- (e) Use the information in the chart to find the y -intercept of the line tangent to the graph of f at the point $(2, 5)$.
- (A) -3 (B) 0 (C) 2 (D) 3 (E) 5

On all the following questions, **show your work**.

5. (20 points) Let $k(x) = 2x^2$.
- (a) Using the definition of derivative, find $k'(x)$
- (b) Evaluate the function found above at $x = 1$ to find $k'(1)$.
- (c) Use the information above to find an equation for the line tangent to the graph of k at the point $(1, k(1))$.
6. (20 points) A division of Moreken Industries manufactures microwave ovens. The daily cost (in dollars) of producing x ovens is given by $C(x) = -0.03x^2 + 120x + 15000$
- (a) What is the actual cost of producing the 201st microwave oven?
- (b) Find the marginal cost function $C'(x)$.
- (c) Find $C'(200)$.
- (d) Find the average cost function $\bar{C}(x)$.