July 21, 2005 **Name** _____

The total number of points available is 122. Throughout this test, **show your work.**

1. (10 points) Consider the parabola $f(x) = 3x^2 + 2x + 2$.

   (a) What is the slope of the line tangent to the graph of f at the point $(0, 2)$?

   (b) Write an equation of this tangent line in the form $y = mx + b$.

2. (12 points) The point $P(3, 19)$ lies on the curve $y = x^2 + x + 7$. If Q is the point $(x, x^2 + x + 7)$, find the slope of the secant line PQ for the following values of x.

   (a) If $x = 3.1$, the slope of PQ is:

   (b) If $x = 3.01$, the slope of PQ is:

   (c) If $x = 2.9$, the slope of PQ is:

   (d) If $x = 2.99$, the slope of PQ is:

   (e) Based on the above results, guess the slope of the tangent line to the curve at $P(3, 19)$.

3. (10 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{2x - 2}$ and let $[a, b] = [1, 3]$. Finally, suppose $M = 1$. Find the number c whose existence is guaranteed by the IVT.

4. (15 points) Let $f(x) = 2/x$.

(a) Construct $\frac{f(3+h)-f(3)}{h}$

(b) Simplify and take the limit of the expression in (a) as h approaches 0 to find $f'(3)$.

(c) Use the information found in (b) to find an equation for the line tangent to the graph of f at the point $(3, 2/3)$.

5. (30 points) Recall that $\sqrt{x}$ is a well-defined real number if and only if $x \geq 0$. Use this fact to find the domain of the function $g(x)$ defined by

$$g(x) = \sqrt{(x-5)(x-3)(x)(x+1)^2(x+4)}.$$

It's important to show all your work, including the test points and the matrix of values of the factors at the test points.

6. (15 points) Let $F(x) = f(x^3)$ and $G(x) = (f(x))^3$. You also know that $a^2 = 10, f(a) = 3, f'(a) = 14, f'(a^3) = 2$.

(a) Find $F'(a)$.

(b) Find $G'(a)$.

7. (30 points) Compute the following derivatives.

(a) Let $f(x) = x^2 + x^{-\frac{2}{3}}$. Find $\frac{d}{dx}f(x)$.

(b) Let $g(x) = \sqrt{x^3 + x + 4}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((3x+1)^2 \cdot (4x^2 - 1))$

(d) Let $f(x) = (2x^2 + 1)^4$. Find $f''(x)$.

(e) Find $\frac{d}{dt}(t^3 + 1/t)^2$.