

October 19, 2006

Name _____

The total number of points available is 148. Throughout this test, **show your work.**

1. (9 points) Let $f(x) = x^3 - 2x - 3$.

(a) Compute $f'(x)$

(b) What is $f'(2)$?

(c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} x + x^3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x^{1/2} & \text{if } x > 1 \end{cases}$$

(a) Is f continuous at $x = 1$?

(b) What is the slope of the line tangent to the graph of f at the point $(4, 4)$?

(c) Find $f'(-3)$

3. (15 points) If a ball is thrown vertically upward from the roof of 212 foot building with a velocity of 48 ft/sec, its height after t seconds is $s(t) = 212 + 48t - 16t^2$.
- (a) What is the height the ball at time $t = 0$?

 - (b) What is the velocity of the ball at the time it reaches its maximum height?

 - (c) At what time is the velocity zero?

 - (d) What is the maximum height the ball reaches?

 - (e) What is the velocity of the ball when it hits the ground (height 0)?
4. (10 points) The cost of producing x units of stuffed alligator toys is $C(x) = -0.003x^2 + 6x + 6000$ for $0 \leq x \leq 1000$.
- (a) Find the marginal cost at the production level of 1000 units.

 - (b) Find the (incremental) cost of producing the 1000th toy.

5. (30 points) Consider the table of values given for the functions $f, f', g,$ and g' :

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = f(x) + g(x)$. Compute $L'(2)$.

(b) Let $U(x) = g \circ g(x)$. Compute $U(1)$.

(c) Let $K(x) = g(x^2) \cdot f(x)$. Compute $K(2)$.

(d) Again, $K(x) = g(x^2) \cdot f(x)$. Compute $K'(2)$.

(e) Let $V(x) = f(f(x))$. Compute $V'(3)$.

(f) Let $W(x) = g(2x) \div f(x)$. Compute $W'(1)$.

(g) Let $Z(x) = f(x^2 + g(x))$. Compute $Z'(1)$.

6. (25 points) Compute the following derivatives. There is no need to simplify except in part (c).

(a) Let $f(x) = (x + \sqrt{1 + x^3})$. Find $\frac{d}{dx}f(x)$.

(b) Let $g(x) = x^3/\sqrt{1 + x^2}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((x + 2) \cdot (2x - 1)^2)$.

(d) Find $\frac{d}{dx}\sqrt{\frac{2x+1}{3x^2-2}}$.

(e) Find $\frac{d}{dt}(t - 1/t^2)^3$.

7. (40 points) Consider the function

$$f(x) = \sqrt{\frac{(x^2 - 1)(3x + 1)}{(2x^2 - 8)(x + 1)}}.$$

Use the Test Interval Technique to find the (implied) domain of $f(x)$.

8. (7 points) Suppose $f(x)$ satisfies $f(3) = 2$ and the line tangent to the graph of f at the point $(3, 2)$ is $2y + 3x = 13$. What is $f'(3)$?