October 25, 2007 **Name** ______________________________

The total number of points available is 143. Throughout this test, **show your work.**

1. (12 points) Let $f(x) = \sqrt{x^2 - x + 3}$.

   (a) Compute $f'(x)$

   (b) What is $f'(3)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} \sqrt{x+3} & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2(x-2)^2 & \text{if } x > 1 \end{cases}$$

   (a) Is f continuous at $x = 1$? Your answer must make clear that you know and understand the definition of continuity. A yes/no correct answer is worth 1 point.

   (b) What is the slope of the line tangent to the graph of f at the point $(8, 72)$?

   (c) Find $f'(-2)$

3. (18 points) If a ball is thrown vertically upward from the roof of 128 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 128 + 64t - 16t^2$.

   (a) What is the height the ball at time $t = 1$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) What is the maximum height the ball reaches?

   (d) After how many seconds is the ball exactly 160 feet above the ground?

   (e) How fast is the ball going the first time it reaches the height 160?

   (f) How fast is the ball going the second time it reaches the height 160?

4. (12 points) The cost of producing x units of stuffed alligator toys is $C(x) = 0.004x^2 + 8x + 6000$.

   (a) Find the marginal cost at the production level of 1000 units.

   (b) What is the marginal average cost function?

   (c) What is $\overline{C}'(500)$? Interpret your answer.

5. (30 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = f(x)/g(x)$. Compute $L'(2)$.

(b) Let $U(x) = f \circ f(x)$. Compute $U'(1)$.

(c) Let $K(x) = \sqrt{f(x)}$. Compute $K'(1)$

(d) Let $V(x) = x^2(g(2x))$. Compute $V'(3)$.

(e) Let $W(x) = [g(x - f(x))]^3$. Compute $W'(4)$.

(f) Let $Z(x) = f(x^2 + g(x))$. Compute $Z'(1)$.

6. (15 points) Compute the following derivatives.

   (a) Let $f(x) = (x + \sqrt{1 + x^3})^4$. Find $\frac{d}{dx} f(x)$.

   (b) Let $g(x) = x^2/(1 + x^2)$. What is $g'(x)$?

   (c) Find $\frac{d}{dx} \sqrt{\frac{2x^3+1}{3x-2}}$.

7. (12 points) Find all critical points of $f(x) = ((x+2)^2 \cdot (2x-1))$. Then identify each critical point as the location of a local maximum, local minimum, or neither.

8. (20 points) Suppose a function f has been differentiated to give

$$f'(x) = (x^2 - 4)(x)(3x^2 - 75)(x - 2).$$

Use the Test Interval Technique on f' to find the sign chart for f'. Then list in interval notation the intervals over which the function f is increasing.