

February 28, 2008

Name _____

The total number of points available is 137. Throughout this test, **show your work.**

1. (15 points) Let $f(x) = \sqrt{x^4 - 3x + 11}$.

(a) Compute $f'(x)$

(b) What is $f'(1)$?

(c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(1, f(1))$.

2. (15 points) For what values of x is the line tangent to the graph of

$$f(x) = (2x + 1)^2(3x - 4)^2$$

parallel to the line $y = 7$?

3. (32 points) The cost of producing widgets is given by $C(x) = 10000 + 50x - 0.003x^2$, $0 \leq x \leq 1000$. The relationship between price and demand for widgets is given by $p = f(x) = -0.04x + 300$, $0 \leq x \leq 7000$, where p is the price in dollars.
- (a) Find the average cost function $\bar{C}(x)$.
 - (b) Find the (incremental) cost of producing the 500th widget.
 - (c) Find the marginal cost function $C'(x)$.
 - (d) What is $C'(500)$?
 - (e) Find the marginal average cost function $\bar{C}'(x)$.
 - (f) Find the revenue function $R(x)$.
 - (g) Find the marginal revenue function $R'(x)$.
 - (h) Find the profit function $P(x)$.
 - (i) Find the marginal profit function $P'(x)$.
 - (j) Find a value of x where the profit function $P(x)$ has a horizontal tangent line.

4. (20 points) Compute the following derivatives.

(a) Let $f(x) = (1 + \sqrt{1 + x^2})^2$. Find $\frac{d}{dx}f(x)$.

(b) Find $\frac{d}{dt}(t^{-3} - \sqrt{t^3})$.

(c) Let $g(x) = x^2/(1 + x^2)$. What is $g'(x)$?

(d) Find $\frac{d}{dx}\sqrt{\frac{2x^2+1}{3x+2}}$.

5. (20 points) Find the domain of the function

$$f(x) = \sqrt{\frac{(x-2)(x+2)(3x-1)}{(3x^2-27)(x-2)}}.$$

Express your answer in interval form.

6. (35 points) Consider the table of values given for the functions f , f' , g , and g' :

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = \frac{x+f(x)}{g(x)}$. Compute $L'(2)$.

(b) Let $U(x) = f \circ f(2x)$. Compute $U'(1)$.

(c) Let $K(x) = g(x^3) + f(x)$. Compute $K'(1)$.

(d) Let $Z(x) = g(2x - f(x))$. Compute $Z'(3)$. Be careful here with the parens. Note that the inside function is $2x - f(x)$.

(e) Let $Q(x) = g(2x) \cdot f(3x)$. Compute $Q'(2)$.

(f) Let $V(x) = f(2 + g(x - 2))$. Compute $V'(2)$.

(g) Let $W(x) = g(x - f(x))$. Compute $W'(5)$.