November 5, 2008 **Name** ________________________________

The total number of points available is 139. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = (2x+3)^2(x-1)^2$.

   (a) Use the product rule to find $f'(x)$.
   Solution: $f'(x) = 2(2x+3)2(x-1)^2 + 2(x-1)(2x+3)^2$

   (b) List the critical points of f.
   Solution: Factor the expression above to get $2(2x+3)(x-1)[2(x-1)+2x+3 = 2(2x+3)(x-1)(4x+1)$, which has value 0 when $x = -3/2, 1, -1/4$

   (c) Construct the sign chart for $f'(x)$.
   Solution: f' is positive on $(-3/2, -1/4)$ and on $(1, \infty)$.

   (d) Write in interval notation the interval(s) over which f is increasing.
   Solution: f is increasing on $(-3/2, -1/4)$ and on $(1, \infty)$

2. (15 points) Consider the function $f(x) = \frac{(2x+3)}{(x-1)^2}$.

   (a) Use the quotient rule to find both $f'(x)$ and $f''(x)$.

   Solution: By the quotient rule $f'(x) = \frac{2(x-1)^2 - 2(x-1)(2x+3)}{(x-1)^4}$ which can be factored to get $f'(x) = \frac{2(x-1)[x-1-(2x+3)]}{(x-1)^4} = \frac{-2x-8}{(x-1)^3}$

   (b) Construct the sign chart for $f''(x)$.

   Solution: By the quotient rule $f''(x) = \frac{-2(x-1)^3 - 3(x-1)^2(-2x-8)}{(x-1)^6} = \frac{4x+26}{(x-1)^4}$, so there is just one zero, $x = -6.5$ and one undefined spot, $x = 1$.

   (c) Write in interval notation the interval(s) over which f is concave upwards.

   Solution: After building the sign chart for f'', we see that f'' is positive on $(-6.5, 1)$ and $(1, \infty)$.

3. (15 points) Consider the function $f(x) = \frac{(2x+3)(x-3)}{x(x-1)}$.

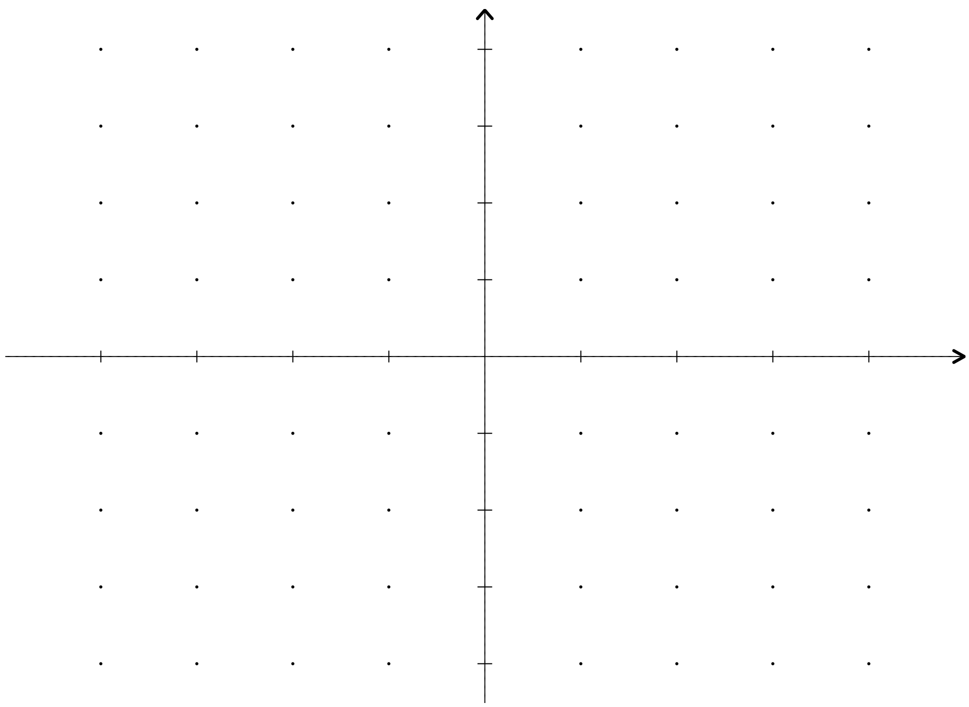
   (a) Build the sign chart for f

   Solution: We have to use all the points where f could change signs, $x = -3/2, 3, 0$, and 1. As expected the signs alternate starting with + at the far left:$+ - + - +$.

   (b) Find the vertical and horizontal asymptotes.

   Solution: The vertical asymptotes are $x = 0$ and $x = 1$, and the horizontal asymptote is $y = 2$.

   (c) Use the information from the first two parts to sketch the graph of f.



4. (10 points) If 1400 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

   Solution: Use $x^2 + 4xy = 1400$ to find y in terms of x. Then $V(x) = x \cdot \frac{1400-x^2}{4}$ after cancelling a pair of x's. Differentiate and set v' to zero to get $\overline{x} = \sqrt{1400/3}$ and finally $V(\overline{x}) = 700/3 \cdot \sqrt{1400/3}$

5. (12 points) A baseball team plays in he stadium that holds 56000 spectators. With the ticket price at \$9 the average attendance has been 23000. When the price dropped to \$8, the average attendance rose to 28000. If $p(x)$ represents the price, in dollars, which will attract x spectators,

   (a) Find the demand function $p(x)$, where x is the number of the spectators. Assume $p(x)$ is linear.

   Solution: We have two points on the linear function $p(23000) = 9$ and $p(28000) = 8$. The slope is $m = \frac{9-8}{23000-28000} = -\frac{1}{5000}$. Using the point slope form, we get $p(x) - 9 = -\frac{1}{5000}(x - 23000)$. Thus, $p(x) = -\frac{1}{5000}(x - 23000) + 9 = -\frac{x}{5000} + \frac{23}{5} \cdot 9$.

   (b) How should be set a ticket price to maximize revenue?

   Solution: The revenue function is the product of x and $p(x)$. Thus $R(x) = xp(x) = -\frac{x^2}{5000} + \frac{68x}{5}$. Differentiating, we get $R'(x) = -2x/5000 + 68/5$. Thus R has just one critical point, $x = \frac{68000}{2} = 34000$ spectators.

6. (6 points) The line $y = 3x - 5$ is tangent to the graph of the function f at the point $(2, 1)$. What is $f'(2)$?

   Solution: $f'(2) = 3$, the slope of the tangent line.

7. (12 points) For what values of x is the tangent line of the graph of

$$f(x) = 2x^3 - 15x^2 - 72x + 12$$

parallel to the line $y = 12x - 17$?

Solution: Since $f'(x) = 6x^2 - 30x - 72 = 6(x^2 - 5x - 12)$, we seek those values of x such that $6(x^2 - 5x - 12) = 12$. Factoring and simplifying yields $x^2 - 5x - 12 - 2 = x^2 - 5x - 14 = 0$ and $(x - 7)(x + 2) = 0$, so the two values of are $x = 7$ and $x = -2$.

8. (12 points) Consider the function $f(x) = x^3 - 5.5x^2 - 4x + 7, \quad -5 \leq x \leq 5$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

Solution: Since $f'(x) = 3x^2 - 11x - 4 = (3x + 1)(x - 4) = 0$ we have the two critical points $x = -1/3$ and $x = 4$. The other two candidates for extrema are the endpoints, -5 and 5. Checking functional values, we have $f(-5) = -235.5, f(-1/3) \approx 7.685$, $f(5) = -25.5$ and $f(4) = -33$. So f has an absolute maximum of about 7.685 at $x = -1/3$ and an absolute minimum of -235.5 at $x = -5$.

9. (12 points) For each function listed below, find all the critical points. Tell whether each critical point gives rise to a local maximum, a local minimum, or neither.

   (a) $f(x) = (x^3 - 8)^2$
   Solution: $f'(x) = 2(x^3 - 8) \cdot 3x^2$, so the critical points are $x = 2$, and $x = 0$. Looking at the sign chart of f', we see that f' does not change signs at $x = 0$, so f does not have an extremum at 0. But f' changes from negative to positive at $x = 2$, so f must have a minimum there.

   (b) $g(x) = (x - 1)^{2/3}$
   Solution: $g'(x) = \frac{2}{3}(x - 1)^{-1/3}$, which means that g has a singular point at $x = 1$. Since f' is negative to the left of 1 and positive to the right, we know f has a minimum at $x = 1$.

10. (15 points) Let $L(x) = 3x - 4$. Of course L is a linear function. For each real number x, the point $(x, y) = (x, 3x - 4)$ belongs to the line. The point $(1, 1)$ does not belong to the line.

(a) Let x denote the number of letters in your first name. Find the distance between $(1, 1)$ and $(x, L(x))$.

Solution:

(b) Let x denote the number of letters in your family name. If this is the same number as in (a), add one to it. Find the distance between $(1, 1)$ and $(x, L(x))$.

Solution:

(c) Find the distance function $D(x)$ that measures the distance from $(1, 1)$ to $(x, L(x))$, where x is arbitrary. The first two parts are samples of function values.

Solution: $D(x) = \sqrt{(x-1)^2 + (3x-4-1)^2} = [(x-1)^2 + (3x-5)^2]^{1/2}$.

(d) Find the derivative $D'(x)$.

Solution: $D'(x) = 1/2[(x-1)^2 + (3x-4-1)^2]^{-1/2} \cdot (2(x-1) + 2(3x-5))$.

(e) Differentiate the square of $D(x)$. This should be much easier to work with.

Solution: The square is $D^2 = (x-1)^2 + (3x-5)^2$ and its derivative is $\frac{dD^2(x)}{dx} = 2(x-1) + 2(3x-5) \cdot 3$ which is zero at $x = 32/20 = 1.6$

(f) Find a critical point of the square of D. Its the same as we would get for D itself.

Solution: $x = 1.6$.

(g) Find the point on the line that is closest to $(1, 1)$.

Solution: The point on the line that is closest is $(1.6, 0.8)$.

11. (10 points) Build a (symbolic representation of a) function f satisfying

    (a) f has zeros at $x = 3$ and $x = -1$.

    (b) f has vertical asymptotes at $x = -4$ and $x = 0$.

    (c) f has $y = 2$ as a horizontal asymptote.

    Solution: $f(x) = \frac{2(x-3)(x+1)}{x(x+4)}$