

March 5, 2009

Name _____

The problems count as marked. The total number of points available is 131. Throughout this test, **show your work.**

1. (12 points) Consider the cubic curve $f(x) = 2x^3 + 3x + 2$.

(a) What is the slope of the line tangent to the graph of f at the point $(0, 2)$?

(b) Write an equation of this tangent line in the form $y = mx + b$.

2. (10 points) Suppose f is a function satisfying $f(2) = 3$ and $f'(2) = -1$. What is the y -intercept of the line tangent to the graph of f at the point $(2, 3)$?

3. (15 points) For what values of x is the line tangent to the graph of

$$f(x) = (2x + 1)^2(3x - 4)^2$$

parallel to the line $y = 7$?

4. (20 points) If a ball is thrown vertically upward from the roof of 256 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 256 + 64t - 16t^2$.
- (a) What is the height the ball at time $t = 1$?

 - (b) What is the velocity of the ball at the time it reaches its maximum height?

 - (c) At what time t does the ball reach its maximum height?

 - (d) What is the maximum height the ball reaches?

 - (e) After how many seconds is the ball exactly 176 feet above the ground?

 - (f) The second derivative $s''(t)$ of the position function, also called the acceleration function, is denoted $a(t)$. Compute $a(t)$. Explain why this function is negative for all values of t .

 - (g) How fast is the ball going the first time it reaches the height 176? Write the answer with the correct units.

 - (h) How fast is the ball going when it hits the ground?

5. (20 points) Let

$$g(x) = (2x^2 - 1)^2(6x).$$

(a) Compute $g'(x)$.

(b) Find the critical points of $g(x)$.

(c) Build the sign chart for $g'(x)$.

(d) Use the sign chart for $g'(x)$ to discuss the nature of each critical point. In other words tell whether each critical point is the location of a local maximum, a local minimum, or neither.

6. (30 points) Consider the table of values given for the functions f , f' , g , and g' :

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $V(x) = f(x) \cdot g(x)$. Compute $V'(5)$.

(b) Let $W(x) = \frac{g(x)}{f(x)}$. Compute $W'(3)$.

(c) Let $L(x) = f(x+1) - g(x)^2$. Compute $L'(2)$.

(d) Let $U(x) = (f \circ g)(2x+1)$. Notice that this is a composition of three functions. Compute $U'(1)$.

(e) Let $K(x) = g(x^3+2)$. Compute $K'(1)$

7. (24 points) Compute the following derivatives.

(a) Let $f(x) = x^2 + x^{-\frac{2}{3}}$. Find $\frac{d}{dx}f(x)$.

(b) Let $g(x) = \sqrt{x^2 + x + 4}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((3x + 1)^2 \cdot (4x^2 - 1))$

(d) Find $\frac{d}{dt} \frac{2t^2 - 3t}{t^2 - 1}$