November 5, 2009 **Name** ________________________________

The problems count as marked. The total number of points available is 155. Throughout this test, **show your work.**

1. (25 points) Let $f(x) = 3x^4 + 4x^3 - 72x^2 + 2$.

   (a) Find the intervals over which f is increasing.

   (b) Find $f(0)$ and use this together with your answer to part (a) to sketch the graph of f.

   (c) Find $f'(0)$ and use this with the information in part (b) to find an equation for the line tangent to the graph of f at the point $(0, f(0))$.

2. (20 points) Suppose the function g has been differentiated twice to get $g''(x) = (x-4)(x+1)(2x+9)$.

   (a) Construct the sign chart for g''.

   (b) Find the intervals over which the function g is concave upwards.

3. (20 points) Let $f(x) = (x^2-4)^{2/3}$. Find $f'(x)$. Find all the critical points and identify each one as the location of a relative Maximum, a relative minimum, or neither (an imposter).

4. (20 points) Compute the following derivatives.

   (a) Let $r(x) = (x^2 - x) \cdot e^{2x-3}$. Find $\frac{d}{dx}r(x)$. Recall that $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$.

   (b) Use the fact that $e^x \neq 0$ for all x to find the critical points of the function r in part (a).

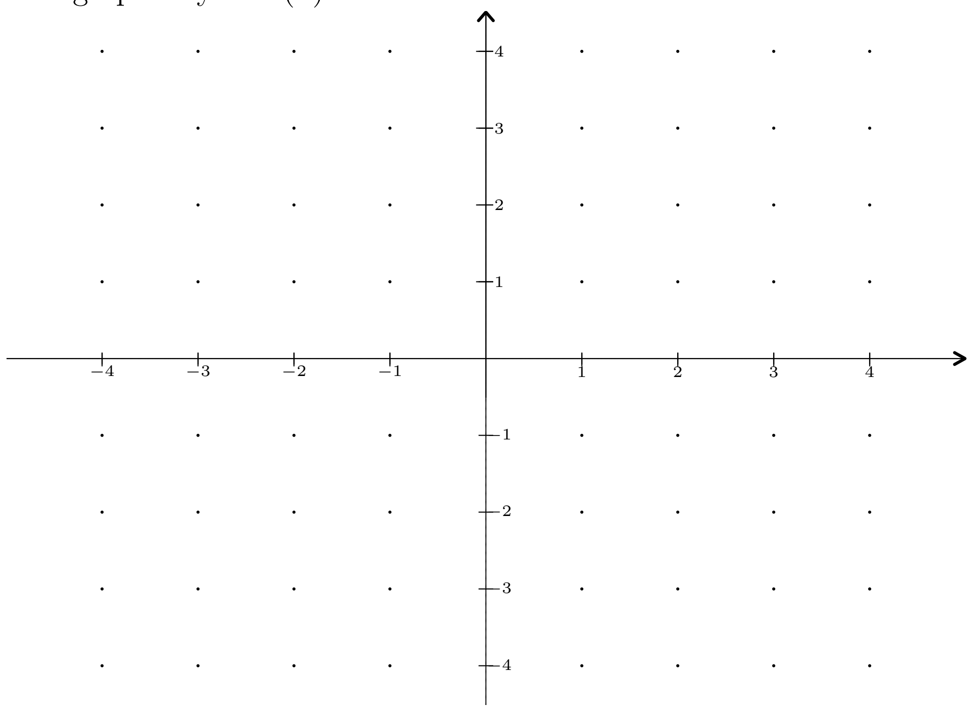
   (c) Let $k(x) = \sqrt{x^3 - 6x^2 + 5x - x^{-1}}$. What is $k'(x)$?

   (d) Let $g(x) = \frac{2x^3+1}{3x-2}$. Find $g'(x)$.

5. (20 points) Find a rational function $r(x)$ that has all the following properties:

   - It has exactly two zeros, $x = -2$ and $x = 3$.
   - It has two vertical asymptotes, $x = 0$ and $x = -3$.
   - It has $y = 2$ as a horizontal asymptote.

   (a) Sketch the graph of your $r(x)$.



   (b) Find a symbolic representation of r.

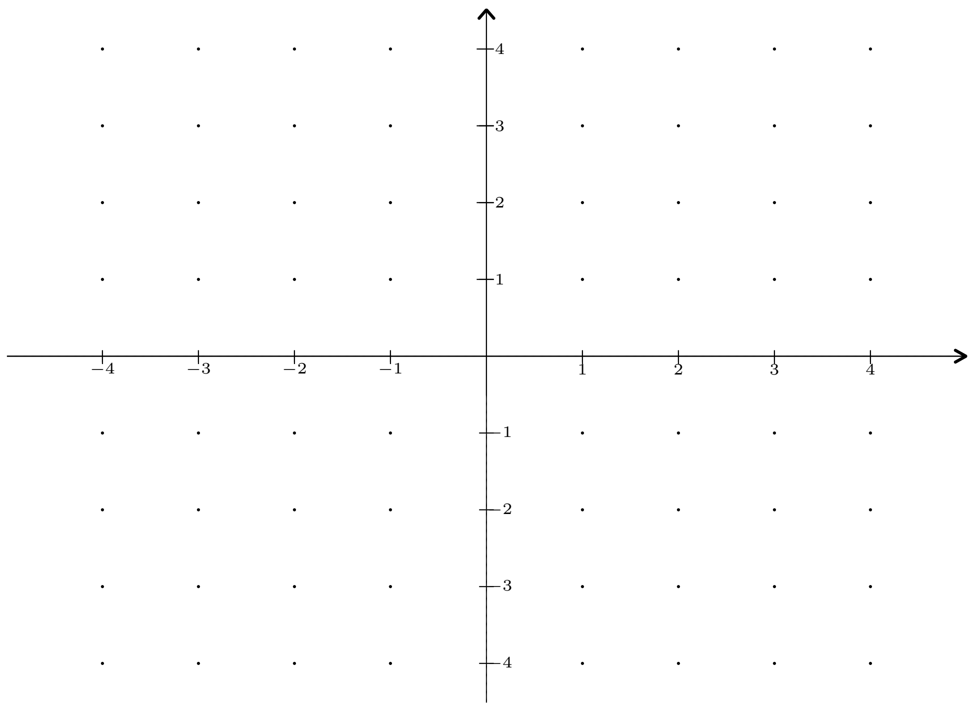
6. (20 points) A baseball team plays in he stadium that holds 60000 spectators. With the ticket price at 12 dollars the average attendance has been 25000. When the price dropped to 10 dollars, the average attendance rose to 40000.

   (a) Find the demand function $p(x)$, where x is the number of the spectators and $p(x)$ is measured in dollars, assuming it is linear. In other words, if the relationship between the price and number of tickets sold is linear, find the price when x tickets are sold.

   (b) How should the ticket price be set to maximize revenue?

7. (10 points) Sketch the graph of the function

$$f(x) = \frac{|\,x-1\,|}{x-1} + \frac{|\,x+3\,|}{x+3}.$$



8. (20 points) A rancher wants to fence in an area of 10 square miles in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?