March 17, 2010 **Name** ____________________

The problems count as marked. The total number of points available is 164. Throughout this test, **show your work.**

1. (24 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. No need to simplify. You must show your work.

   (a) Let $F(x) = (x^2 - 3x + 1)(x^3 - 2x + 5)$

   (b) $G(x) = \dfrac{2x^4 - 3x + 1}{x^2 - x + 3}$

   (c) $K(x) = (x^2 - 3)^{17}$

   (d) $H(x) = \sqrt{(3x + 1)^4 - 7}$.

2. (10 points) The line tangent to the graph of $g(x)$ at the point $(4, 7)$ has a y-intercept of 9. What is $g'(4)$?

3. (10 points) Find a point on the graph of $h(x) = x^3 - 6x^2 + 9x$ where the tangent line is horizontal. There are two such points on the graph of $h(x)$.

4. (25 points) A stone is thrown upwards from the top of a 200 foot high building in such a way that its height is given by

$$s(t) = -16t^2 + 128t + 200$$

feet, where t is measured in seconds.

(a) Find the height of the stone for each value of t listed:

i. $t = 0$

ii. $t = 1$

iii. $t = 2$

iv. $t = 2.1$

v. $t = 2.01$

(b) How far did the stone travel during the first two seconds of its flight?

(c) What was the average speed of the stone during those first two seconds?

(d) How far did the stone travel during the time interval $[2, 2.1]$ and what was its average velocity during that time?

(e) How far did the stone travel during the time interval $[2, 2.01]$ and what was its average velocity during that time?

(f) What is $s'(t)$? What is the relation between $s'(2)$ and the numbers you found in parts (d) and (e).

(g) What is the maximum height attained by the stone?

5. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	1	4

(a) $Q(x) = f(x)/g(x)$. Find $Q'(5)$.

(b) Let $H(x) = f(x) \cdot (g(x) + 1)$. Compute $H'(4)$.

(c) Let $W(x) = f(g(x) + 1)$. Compute $W'(5)$.

(d) Let $L(x) = g(\frac{1}{x} + 1)$. Compute $L'(1)$.

(e) Let $U(x) = \sqrt{g(2x)}$. Compute $U'(3)$.

(f) Let $Z(x) = g(2x - f(x))$. Compute $Z'(4)$.

6. (30 points) The cost of producing widgets is given by $C(x) = 10000 + 50x - 0.003x^2, \quad 0 \leq x \leq 1000$. The relationship between price and demand for widgets is given by $p = f(x) = -0.04x + 300, \quad 0 \leq x \leq 7000$, where p is the price in dollars.

   (a) Find the average cost function $\overline{C}(x)$.

   (b) Find the (incremental) cost of producing the 500^{th} widget.

   (c) Find the marginal cost function $C'(x)$.

   (d) What is $C'(500)$?

   (e) Find the marginal average cost function $\overline{C}'(x)$.

   (f) Find the revenue function $R(x)$.

   (g) Find the marginal revenue function $R'(x)$.

   (h) Find the profit function $P(x)$.

   (i) Find the marginal profit function $P'(x)$.

   (j) Find a value of x where the profit function $P(x)$ has a horizontal tangent line.

7. (30 points) Let $g(x) = (x^2 - 4)^2(2x + 1)^2$. Using the chain and product rules, we can differentiate $g(x)$ to get to find

$$g'(x) = 2(x^2 - 4)(2x)(2x + 1)^2 + 2(2x + 1) \cdot 2(x^2 - 4)^2.$$

   (a) Find all the x-intercepts (the zeros) of $g'(x)$. That is, find the critical points of g.

   (b) Build the sigh chart for $g'(x)$.

   (c) Use the sign chart for $g'(x)$ to classify each critical point of g found in part (a) as the location of (i) a local minimum, (ii) a local maximum, or (iii) an imposter.