November 4, 2010 **Name** _____

The problems count as marked. The total number of points available is 145. Throughout this test, **show your work.**

1. (10 points) Suppose f and g are functions for which both $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$. Which of the following is true? Circle your answer.

   (A) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = 0$ **(B)** $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ does not exist **(C)** $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \infty$

   (D) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ can be any real number **(E)** $\lim_{x\to a} \dfrac{f(x)}{g(x)} = 1$

2. (20 points) Let $f(x) = \sqrt{x^4 - 3x + 11}$.

   (a) Compute $f'(x)$

   (b) What is $f'(1)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(1, f(1))$.

3. (20 points) Use calculus to find all relative max and min and also all asymptotes of the function $g(x) = 4x + 16/x$.

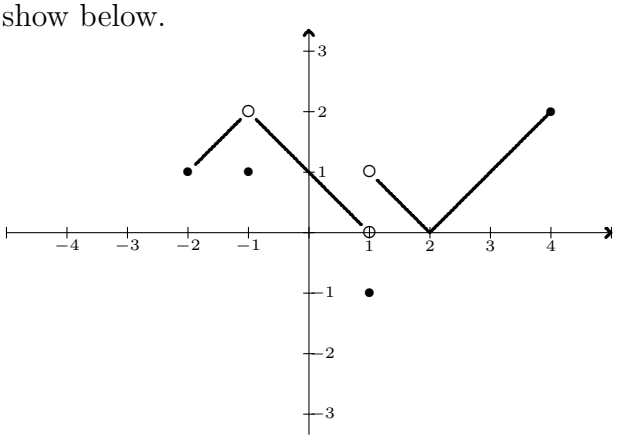
   Then, use calculus to discuss the concavity of $g(x)$.

4. (20 points) Let
$$H(x) = (2x + 1)^2(3x - 4)^2.$$

   (a) For what values of x is the line tangent to the graph of $H(x)$ parallel to the line $y = 7$?

   (b) Find the intervals over which $H(x)$ is increasing.

5. (20 points) Find the symbolic representation of the function G whose graph is show below.



As a hint, the function needs six clauses, as shown.

$$G(x) = \begin{cases} \\ \\ \\ \\ \\ \\ \end{cases} \begin{array}{l} \\ \text{if } -2 \le x < -1 \\ \text{if } x = -1 \\ \text{if } -1 < x < 1 \\ \text{if } x = 1 \\ \text{if } 1 < x < 2 \\ \text{if } 2 \le x \le 4 \end{array}$$

6. (20 points) Use calculus to find the point $P = (u, v)$ on the line $2x + 3y = 7$ that is closest to the origin $(0, 0)$. Then use geometry to check your answer. Is the slope of the line $y = (v/u)x$ right? Write a complete sentence about your reasoning.

7. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = (f(x) + g(x))^2$. Compute $L'(2)$.

(b) Let $U(x) = f \circ f \circ f(x)$. Compute $U'(1)$.

(c) Let $K(x) = g(x) + f(x^2)$. Compute $K'(2)$

(d) Let $Z(x) = f(x) \div g(x)$. Compute $Z'(3)$.

(e) Let $Q(x) = g(3x) \cdot f(2x)$. Compute $Q'(2)$.