

October 27, 2011

Name _____

The problems count as marked. The total number of points available is 148. Throughout this test, **show your work.**

1. (9 points) Let $f(x) = x^4 - 1/x - 3$.

(a) Compute $f'(x)$

(b) What is $f'(1)$?

(c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(1, f(1))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} x + x^3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x^{3/2} & \text{if } x > 1 \end{cases}$$

(a) Is f continuous at $x = 1$?

(b) What is the slope of the line tangent to the graph of f at the point $(4, 16)$?

(c) Find $f'(-3)$

3. (10 points) The cost of producing x units of stuffed alligator toys is $C(x) = -0.003x^2 + 6x + 6000$ for $0 \leq x \leq 1000$.

(a) Find the marginal cost at the production level of 1000 units.

(b) Find the (incremental) cost of producing the 1000th toy.

4. (15 points) Consider the function $f(x) = x^3 - 6x$ defined on the interval $-2 \leq x \leq 3$.

(a) Find the critical points of f .

(b) Find the absolute minimum of f and the x -value where it occurs.

(c) Find the absolute maximum of f and the x -value where it occurs.

5. (30 points) Consider the table of values given for the functions f , f' , g , and g' :

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	3	4	2	3
3	1	2	5	3
4	3	5	2	5
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = f(x) \cdot g(x)$. Compute $L'(5)$.

(b) Let $U(x) = f \circ f(x)$. Compute $U'(4)$.

(c) Let $K(x) = (g(x) + f(x))^3$. Compute $K(2)$.

(d) Again, $K(x) = (g(x) + f(x))^3$. Compute $K'(2)$.

(e) Let $V(x) = f(x^2) \div g(x)$. Compute $V'(2)$.

(f) Let $Z(x) = g(x^2 + f(x))$. Compute $Z'(1)$.

6. (10 points) Compute the following derivatives.

(a) Let $f(x) = x + \sqrt{1 + x^3}$. Find $\frac{d}{dx}f(x)$.

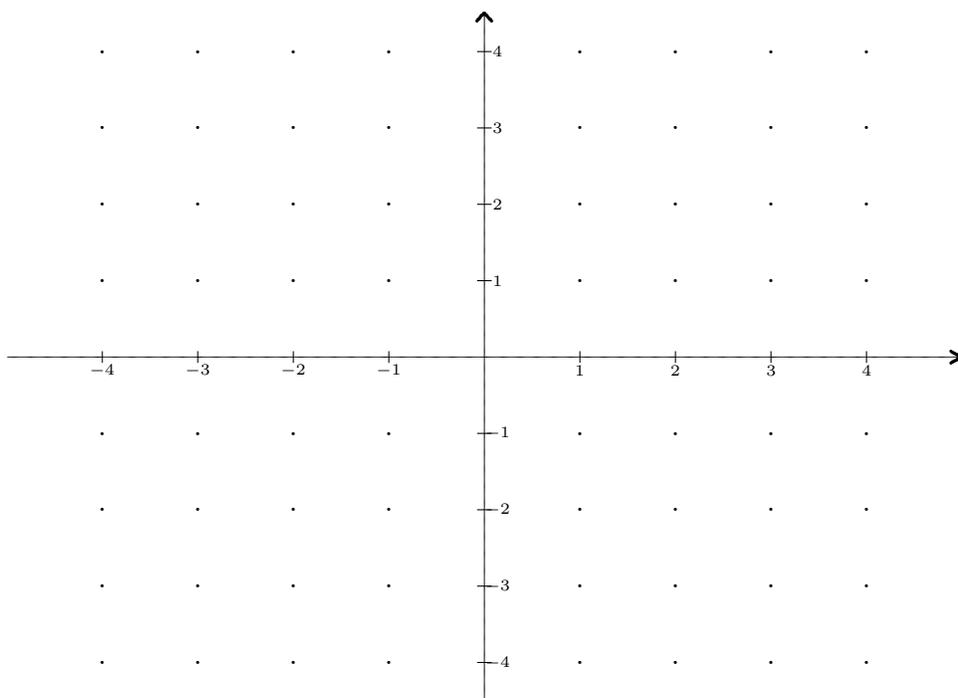
(b) Let $g(x) = \frac{x^3}{x^2+1}$. What is $g'(x)$?

7. (10 points) Find two critical points of $h(x) = (x + 2) \cdot (2x - 1)^2$.

8. (30 points) Consider the function

$$r(x) = \frac{(x^2 - 1)(3x + 1)}{(2x^2 - 8)(x + 1)}.$$

Use the Test Interval Technique to find the sign chart of $r(x)$. Find the horizontal and vertical asymptotes, and sketch the graph of r . Your graph must be consistent with the information you find in the sign chart.



9. (7 points) Suppose $f(x)$ satisfies $f(3) = 2$ and the line tangent to the graph of f at the point $(3, 2)$ is $2y + 3x = 13$. What is $f'(3)$?

10. (15 points) Consider the function $h(x) = x^4 + 2x^3 - 12x^2 + 60x$. Find the intervals over which h is concave upwards. Make clear which function you're building the sign chart for and what the test points are.