February 27, 2013 **Name** ________________________________

The total number of points available is 151. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (12 points) Let $H(x) = \sqrt{x^2 - 2x + 4}$.

   (a) Find two functions, f and g whose composition $f \circ g$ is H, and use the chain rule to find $H'(x)$

   (b) What is $H'(2)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of H at the point $(2, H(2))$.

2. (10 points) Solve the inequality $x^2 - 13x + 14 \leq 2$. Write your answer in interval notation.

3. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} \sqrt{x+8} & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 3(x-2)^2 & \text{if } x > 1 \end{cases}$$

   (a) Is f continuous at $x = 1$? Your answer must make clear that you know and understand the definition of continuity. A yes/no correct answer is worth 1 point.

   (b) What is the slope of the line tangent to the graph of f at the point $(8, 108)$?

   (c) Find $f'(-2)$

4. (12 points) Compute each of the following derivatives.

   (a) Let $f(x) = (2x+1)^2(x^2+x-1)$. Find $f'(x)$.

   (b) Let $g(x) = \frac{x^2+x-1}{x^2+x+1}$. Find $g'(x)$.

5. (18 points) If a ball is thrown vertically upward from the roof of 128 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 128 + 64t - 16t^2$.

   (a) What is the height the ball at time $t = 1$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) What is the maximum height the ball reaches?

   (d) After how many seconds is the ball exactly 160 feet above the ground?

   (e) How fast is the ball going the first time it reaches the height 160?

   (f) How fast is the ball going the second time it reaches the height 160?

6. (12 points) The cost of producing x units of stuffed alligator toys is $C(x) = 0.004x^2 + 4x + 6000$.

   (a) Find the marginal cost at the production level of 1000 units.

   (b) What is the marginal average cost function?

   (c) What is $\overline{C}'(500)$? Interpret your answer. In particular, what does the sign of $\overline{C}'$ at $x = 500$ tell you?

7. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = (f(x) + g(x))^2$. Compute $L(2)$ and $L'(2)$.

(b) Let $U(x) = f \circ f \circ f(x)$. Compute $U(1)$ and $U'(1)$.

(c) Let $K(x) = g(x) + f(x^2)$. Compute $K(2)$ and $K'(2)$

(d) Let $Z(x) = 1/g(2x)$. Compute $Z(3)$ and $Z'(3)$.

(e) Let $Q(x) = g(3x) + f(2x)$. Compute $Q(2)$ and $Q'(2)$.

8. (20 points) Find all critical points of $H(x) = (x+2)^3 \cdot (x^2-1)^2$. Then identify each critical point as the location of a local maximum, local minimum, or neither.

9. (20 points) The purpose of this problem is to show how we can prove the power rule when the exponent is not a positive integer. In class we showed that when n is a positive integer,

$$\frac{d}{dx}x^n = nx^{n-1}.$$

But our proof does not work for fractional exponents. Let $g(x) = x^{1/4}$. We want to prove that $g'(x) = \frac{1}{4}x^{-3/4}$. To accomplish this, construct a function $f(x)$ so that the composition $f \circ g(x)$ can be differentiated using just the power rule with positive integer exponents. Several choices will work here. This part of the problem is worth 6 points. Let $h(x) = f \circ g(x)$. Then use the chain rule to write $h'(x) = f'(g(x)) \cdot g'(x)$. You can find both $h'(x)$ and $f'(x)$ easily, so you can solve for $g'(x)$. Do this to get the desired conclusion

$$g'(x) = \frac{1}{4}x^{-3/4}.$$