

October 24, 2013

Name _____

The problems count as marked. The total number of points available is 171. Throughout this test, **show your work**. Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (36 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. Find the critical points for each function and the intervals over which the function is increasing. You must show your work.

(a) Let $F(x) = (2x + 8)(4x - 6)$

(b) $G(x) = \frac{x^2 - 3x + 15/2}{2x - 1}$

(c) $K(x) = (x^2 - 4)^{18}$

2. (10 points) The line tangent to the graph of $g(x)$ at the point $(4, 6)$ has a y -intercept of 9. What is $g'(4)$?
3. (10 points) Find all the points (x, y) on the graph of $h(x) = 2x^2 - 4x$ where the tangent line has a slope equal to 5.

4. (15 points) Show that the function $f(x) = 7\sqrt{x} - x^3$ has a critical point in the interval $[1, 4]$. Note that both f and f' are continuous on the interval $[1, 4]$.
5. (10 points) The line tangent to the graph of a function f at the point $(2, 9)$ on the graph also goes through the point $(0, 7)$. What is $f'(2)$?
6. (10 points) Find an equation for the line tangent to the graph of $f(x) = x^2 - 3x$ at the point $(2, -2)$?
7. (15 points) Discuss the concavity of the function $r(x) = \frac{2x-3}{x+1}$.

8. (35 points) Consider the table of values given for the functions f , f' , g , and g' :

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	1	4

- (a) $Q(x) = f(x)/g(x)$. Find $Q(5)$ and $Q'(5)$.
- (b) Let $H(x) = f(x) \cdot g(x + 1)$. Compute $H(4)$ and $H'(4)$.
- (c) Let $W(x) = f(g(x) + 1)$. Compute $W(5)$ and $W'(5)$.
- (d) Let $L(x) = g(\frac{1}{x} + 1)$. Compute $L(1)$ and $L'(1)$.
- (e) Let $U(x) = g(x^2 - 2) + f(x)$. Compute $U(2)$ and $U'(2)$.
- (f) Let $Z(x) = g(2x - f(x))$. Compute $Z(4)$ and $Z'(4)$.

9. (30 points) Consider the function

$$r(x) = \frac{(x^2 - 4)(6x)}{(3x - 6)(x + 1)(x - 3)}.$$

Use the Test Interval Technique to find the sign chart of $r(x)$. Find the zeros and the horizontal and vertical asymptotes, and sketch the graph of r . Your graph must be consistent with the information you find in the sign chart.

