

April 27, 2006

Name _____

The total number of points available is 135. Throughout this test, **show your work**.

1. (8 points) How long does it take a 6% investment, compounded continuously, to triple in value?

2. (20 points) Consider the function $f(x) = e^{2x^3 - 6x}$.
 - (a) Compute $f'(x)$.

 - (b) Find the critical points of f .

 - (c) Find the intervals over which f is increasing.

 - (d) Compute $f''(x)$ and discuss the concavity of f . You will have to use your graphing calculator for this part of the problem.

3. (15 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 110e^{-0.02t}$$

where $m(t)$ is measured in grams.

- (a) Find the mass at time $t = 0$.

- (b) How much of the mass remains after 20 years?

- (c) At what rate is the mass declining after 10 years?

4. (10 points) The population of the world in 1987 was 5 billion and the relative growth rate was estimated at 2 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2007.

5. (16 points) A study finds that the average student taking advanced shorthand progresses according to the function

$$Q(t) = 120(1 - e^{-0.05t}) + 60, \quad (0 \leq t \leq 20),$$

where $Q(t)$ measures the number of words (per minute) of dictation that the student can take in machine shorthand after t weeks in the twenty-week course. Sketch the graph of Q and answer the following questions:

- (a) What is the beginning shorthand speed for the average student?
- (b) What shorthand speed does the average student attain halfway through the course?
- (c) How many words per minute can the average student take at the end of the course.
- (d) What is the rate of change of the speed after exactly 5 weeks in the course?
6. (12 points) Let $f(x) = 3 \ln(x^2 - 3)$. Notice that the domain of f includes the point $x = 2$, since $2^2 - 3 = 1 > 0$.
- (a) Find $f'(x)$.
- (b) Find $f'(2)$.
- (c) Find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

7. (16 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the difference between the object's temperature and that of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212^\circ F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $70^\circ F$, A and k are constants, and t is expressed in minutes.

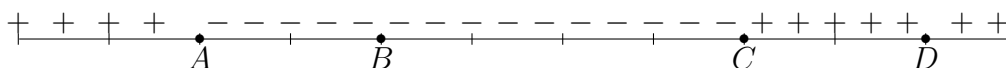
- (a) What is the value of A ?
- (b) Suppose that after exactly 20 minutes, the temperature of the coffee is $186.6^\circ F$. What is the value of k ?
- (c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80^\circ F$.
- (d) Find the rate at which the object is cooling after $t = 20$ minutes.
8. (10 points) Suppose the function $f(x)$ has been differentiated twice to get $f''(x) = (x - 3)(x + 2)(x + 4)$. Find the intervals over which $f(x)$ is concave upward.

9. (12 points) Find the critical points of each function.

(a) $f(x) = (x^2 - 4)^2(2x - 3)^2$

(b) $g(x) = (x^2 - 16)^{2/3}$

10. (16 points) Given below is a sign chart for the derivative $f'(x)$ of a function.



(a) For each of the stationary points A, B, C and D tell whether $f(x)$ has a relative maximum, relative minimum, or neither at the point.

(b) Suppose $f(x)$ is a polynomial function with critical points A, B, C and D . Sketch a function on the coordinate system below that could have a derivative whose sign chart is the one given.

