April 23, 2014 **Name** ______________________________

The problems count as marked. The total number of points available is 172. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (10 points) Find an equation for the line tangent to the graph of $y = \ln(x^4 + 1)$ at the point $(1, \ln(2))$.

2. (20 points) A function $g(x)$ has been differentiated to get

$$g'(x) = 2(x-3)^2 - 8.$$

   (a) Find the interval(s) over which $g'(x)$ is increasing.

   (b) Find the interval(s) over which $g(x)$ is increasing.

   (c) Find the interval(s) over which $g(x)$ is concave upwards.

3. (15 points) Consider the function $f(x) = x^3 - 9x^2 + 24x$ on the interval $[0, 5]$.

   (a) What is the largest value of f on its domain? In other words, find the absolute maximum of f over $[0, 5]$.

   (b) What is the smallest value of f on its domain? In other words, find the absolute minimum of f over $[0, 5]$.

4. (15 points) Consider the function $f(x) = \ln[(2x - 13)(3x - 4)^3\sqrt{x^2 + 3}]$.

   (a) Recall that $\ln(x)$ is defined precisely when $x > 0$. Find the domain of f.

   (b) Let $g(x) = (2x - 13)(3x - 4)^3\sqrt{x^2 + 3}$. Use logarithmic differentiation to find g'. Find a decimal representation of $g'(1)$.

5. (20 points) Find all the critical points for each of the functions listed below.

   (a) $T(x) = 4(x^2 + 9)^{1/2} + 22 - 2x$.

   (b) $f(x) = \ln(2x + 17) - 2x$.

   (c) $g(x) = e^{x^2 - 4x}$.

   (d) $h(x) = (x^2 - 4)^{2/3}$.

6. (20 points) A botanist conjectures that the height of a certain type of pine tree can be modeled by a learning curve. To test his conjecture, he plants a 2 foot tall tree. He knows that eventually the tree will grow to 40 feet tall, its maximum height. Suppose that after one year, the tree is 4 feet tall.

   (a) What does the model predict for the height of the tree after two years.

   (b) How many inches does the tree grow during the fourth year?

   (c) What is the instantaneous rate of growth at $t = 3.5$ years.

   (d) Describe the connection between the two answers (b) and (c).

7. (30 points) Suppose we know that the function f has been differentiated and that $f'(x) = 2x(x^2 - 3)^4$. Also, the point $(2, 1/5)$ belongs to the graph of f.

   (a) Find an equation for the line tangent to the graph of f at the point $(2, 1/5)$.

   (b) Find $f(1)$. Hint: f is an antiderivative of f'.

   (c) Find the area of the region R bounded above by the graph of $f'(x)$, below, by the x-axis and on the sides by the lines $x = 0$ and $x = 1$.

8. (42 points) Compute each of the following integrals

   (a) $\displaystyle\int_1^2 \frac{(4x-5)^2}{x}\ dx$

   (b) $\displaystyle\int_0^1 \frac{d}{dx}(x^3-2x^2+7)\ dx$

   (c) $\displaystyle\int_1^4 3x^2e^{x^3}\ dx$

   (d) $\displaystyle\int_2^3 \frac{x^3+2x^2-x}{x}\ dx$

   (e) $\displaystyle\int_1^3 \frac{2x+3}{x^2+3x-3}\ dx$

   (f) $\displaystyle\int_{-1}^1 6x^5(x^6+3)^7\ dx$

   (g) $\displaystyle\int_1^2 (x-1)^5x\ dx$