Math 1120 **Calculus** **Test 3**

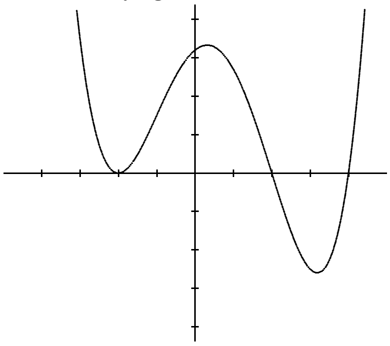
August 2, 1999 Your name ______________________

The multiple choice problems count five points each.

1. Suppose the line tangent to the graph of a function f at the point $(3, 2)$ has y-intercept 8. What is $f'(3)$?

   (A) $\boxed{-2}$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2. Questions (a) through (d) refer to the graph of the fourth degree polynomial function f given below.



   (a) The number of roots of $f''(x) = 0$ is

   (A) 0 **(B)** 1 **(C)** $\boxed{2}$ **(D)** 3 **(E)** 4

   (b) The number of roots of $f'(x) = 1$ is

   (A) 0 **(B)** 1 **(C)** 2 **(D)** $\boxed{3}$ **(E)** 4

   (c) The number of roots of $f(x) = 2$ is

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** $\boxed{4}$

   (d) A good estimate of $f'(1)$ is

   (A) $\boxed{-2}$ **(B)** 0 **(C)** 1 **(D)** 1.8 **(E)** 3.2

On all the following questions, **show your work.**

3. (20 points) Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3 - 4x^2 - x + 4$ on the interval $-2 \le x \le 6$.

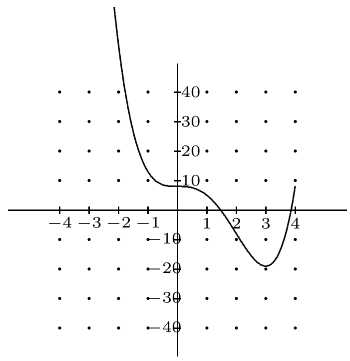
First find the critical points: $f'(x) = 3x^2 - 8x - 1$ does not factor, so we must use the quadratic formula to solve $f'(x) = 0$. So, $a = 3, b = -8$, and $c = -1$ and we get the two roots

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-1)}}{2 \cdot 3}.$$

Use the calculator to get the stationary points $x_1 \approx -0.119$ and $x_2 \approx 2.786$. Compare $f(-2) = -18, f(6) = 70, f(x_1) \approx 4.06$, and $f(x_2) \approx -8.208$. Thus the absolute maximum value is 70 and the absolute minimum value is -18.

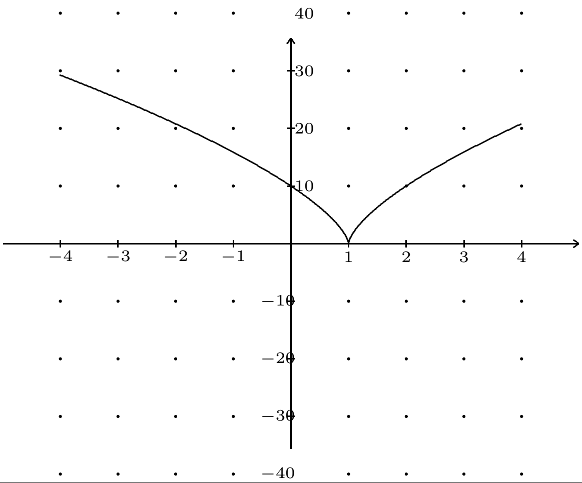
4. (20 points) Find the intervals over which each of the functions is increasing and over which each is decreasing. Sketch the graph on the axes provided.

(a) $g(x) = x^4 - 4x^3 + 8.$



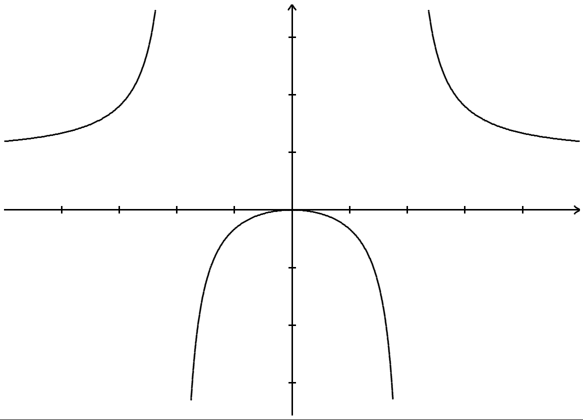
> The function has two critical points, $x = 0$ and $x = 3$. Use the test interval technique to see that the function $g'(x)$ is positive over the interval $(3, \infty)$ and negative over both $(-\infty, 0)$ and $(0, 3)$. Notice that g levels out instantaneously at 0 but continues to decrease immediately. Therefore g is decreasing over the entire interval $(-\infty, 3)$ and increasing over $(3, \infty)$.

(b) $f(x) = (x-1)^{\frac{2}{3}}$.



The function f has one critical point, a singular point at $x = 1$ since its derivative is $f'(x) = 2/3(x-1)^{-1/3}$ which is undefined at 1. Use the test interval technique to see that the function $f'(x)$ is positive over the interval $(1, \infty)$ and negative over $(-\infty, 1)$. Therefore f decreases over the entire interval $(-\infty, 1)$ and increases over $(1, \infty)$.

(c) $h(x) = \frac{x^2}{x^2-4}$.



The function h has derivative $h'(x) = \frac{-8x}{(x^2-4)^2}$ and h has three critical points, two of which are singular points ($x = 2$ and $x = -2$ and one stationary point ($x = 0$). Use the test interval technique to see that the function $h'(x)$ is positive over the intervals $(-\infty, -2)$ and $(-2, 0)$ negative over $(0, 2)$ and $(2, \infty)$. Also, it has a horizontal asymptote of $y = 1$. Therefore h increases over the both intervals $(-\infty, -2)$ and $(-2, 0)$ while it decreases over $(0, 2)$ and $(2, \infty)$.

5. (10 points) Solve each of the equations below for x in terms of the other letters.

   (a) $a^x b^x = (1-a)(2-b)$

   Take logs of both sides to get
   $$x = \frac{\log(1-a)(2-b)}{\log ab},$$
   where the log can have any base (as long as both the numbers $(1-a)(2-b)$ and ab are positive.

   (b) $\frac{a}{1+b^x} = b^4$

   Take log base b of both sides to get
   $$x = \log_b\left(\frac{a}{b^4} - 1\right).$$
   again assuming all the quantities in question are positive.

6. (15 points) Consider the rational function
   $$f(x) = \frac{(4x^2-3)(x-2)}{(x^2-4)(2x+3)}.$$

   (a) Find the horizontal asymptotes.

   Both numerator and denominator have degree 3, so the HA is just the ratio of the coefficients of x^3, which here is $y = 4/2 = 2$.

   (b) Find the vertical asymptotes.

   We look for zeros of the denominator that are not zeros of the numerator, which yields $x = -2$ and $x = -3/2$.

   (c) Compute $\lim_{x\to-\infty} f(x)$. $\boxed{\lim_{x\to-\infty} f(x) = 2.}$

7. (20 points) An open box with a square base has a volume of 54 cubic inches. If material for all five sides (four vertical sides and the square bottom) cost 3 cents per square inch, what is the minimum material cost for the box?

The volume equation is $V = x^2y = 54$ cubic inches, where x is the side of the square base and y is the height. The cost function is $C = 3(x^2 + 4xy)$, which transforms into $C(x) = 3(x^2 + 4x \cdot \frac{54}{x^2})$ when we solve the volume equation for y and replace that value by its equivalent in terms of x. Then $C'(x) = 6x - 648/x^2$, which is zero when $x = 108^{1/3} \approx 4.762$. It follows that the minimum cost is $C(108^{1/3}) \approx 204$ cents.

8. (20 points) Optimal Charter Flight Fare. If exactly 200 people sign up for a charter flight, the agency charges \$300. However, if more than 200 sign up, the agency reduces the fare by \$0.80 for each additional person.

   (a) Let x denote the number of passengers beyond 200. Construct the revenue function $R(x)$.

   $\boxed{R(x) = (200 + x)(300 - 0.8x).}$

   (b) Find all the critical points of your revenue function.

   $\boxed{R'(x) = 300 - 0.8x + (200 + x)(-0.8) = 140 - 1.6x.}$

   (c) What number of passengers results in the maximum revenue?

   $\boxed{x = 87.5\text{, so maximum profits are attained when there are 287.5 passengers.}}$

   (d) What is the maximum revenue?

   $\boxed{R(287) = R(288) = 66,124.8}$