

March 21, 2001

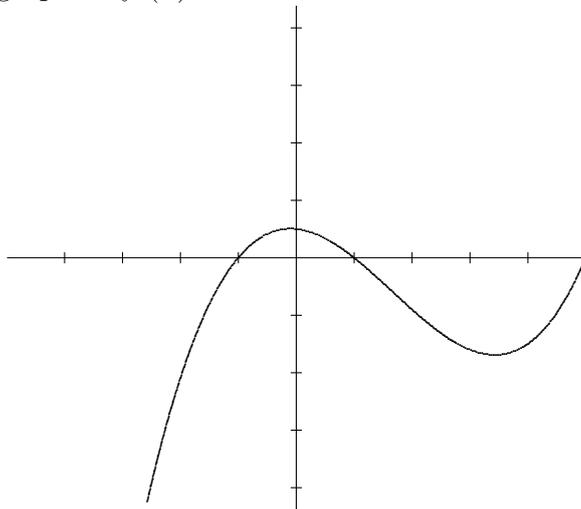
Name _____

The first 11 problems are true-false problems that count 3 points each. The rest are counted as marked. The total value of the test is 125.

True-false section. Circle the correct choice. You do not need to show your work on these problems.

1. True or false. If f and g are differentiable and a and b are constants, then $\frac{d}{dx}[af(x) + bg(x)] = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$.
2. True or false. If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.
3. True or false. If $f''(x) < 0$ on the interval (a, c) and $f''(x) > 0$ on the interval (c, b) , then the point $(c, f(c))$ is a point of inflection of f .
4. True or false. If $f(a) < 0$, $f(b) > 0$, and $f'(x) > 0$ for each x in (a, b) , then there is one and only one number c in (a, b) such that $f(c) = 0$.
5. True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of f .
6. True or false. If $f'(c) = 0$, then f has a relative maximum or a relative minimum at $x = c$.
7. True or false. If f has a relative maximum or a relative minimum at $x = c$, then $f'(c) = 0$.
8. True or false. If $f'(c) = 0$ and $f''(c) < 0$, then f has a relative maximum at $x = c$.
9. True or false. If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.
10. True or false. If f and g are differentiable, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)}{g'(x)}$.
11. True or false. If f and g are differentiable and $h(x) = f \circ g$, then $h'(x) = f[g(x)]g'(x)$.
12. (12 points) Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3 - 4x^2 - x + 4$ on the interval $-2 \leq x \leq 6$.

13. (12 points) Let f be the function whose graph is shown below. On the same axes, plot the graph of $f'(x)$.



14. (12 points) Find the interval(s) where $f(x) = x^3 - 6x^2 - 4x + 8$ is increasing.

15. (12 points) Find the relative maxima and relative minima, if any, of $g(x) = x^2 + \frac{16}{x^2}$.

16. (12 points) Let $f(x) = x^4 + 2x^3 - 12x^2 + 6x$.

(a) Find the interval(s) where f is concave upward and the interval(s) where f is concave downward. Use the Test Interval technique to determine the places where f'' is positive and where it is negative.

(b) Find the inflection points of f , if there are any.

17. (12 points) Consider the rational function

$$f(x) = \frac{(2x^2 - 3)(x - 2)}{(x^2 - 4)(x + 1)}.$$

(a) Find the horizontal asymptotes.

(b) Find the vertical asymptotes.

(c) Compute $\lim_{x \rightarrow -\infty} f(x)$.

On all the following questions, **show your work.**

18. (20 points) The quantity demanded per month, x of a certain brand of electric shavers is related to the price, p , per shaver by the equation $p = -0.1x + 10,000$ ($0 < x < 20,000$), where p is measured in dollars. The total monthly cost for manufacturing the shavers is given by $C(x) = 0.00002x^3 - 0.4x^2 + 10,000x + 20,000$. Construct the revenue function, $R(x)$. How is the profit related to revenue and cost? Find $P'(x)$, where $P(x)$ denotes the profit function. How many shavers should be produced per month in order to maximize the company's profit? What is the maximum profit?