November 15, 2001 Your name ____________________________

The multiple choice count 5 points each and the true-false problems count 3 points each. In the multiple choice section, circle the correct choice (or choices). You must show your work on the other problems. The total number of points available is 122.

1. True-false questions. Circle the appropriate word, true or false.

   (a) True or false. Every continuous function f on an interval $[a, b]$ must have an absolute maximum somewhere in the interval. Ie, there exists a point $c \in [a, b]$ such that $f(c) \geq f(x)$ for all $x \in [a, b]$.

   (b) True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of f.

   (c) True or false. If $f'(c) = 0$, then f has a relative maximum or a relative minimum at $x = c$.

   (d) True or false. If f has a relative maximum or a relative minimum at $x = c$, then $f'(c) = 0$.

   (e) True or false. If $f'(c) = 0$ and $f''(c) < 0$, then f has a relative maximum at $x = c$.

2. A stone is thrown straight upward from the roof of an 80-foot high building. The height in feet of the stone at any time t (in seconds) is given by $h(t) = -16t^2 + 64t + 80$.

   (a) How many seconds elapse before the stone hits the ground?

   (A) 2 seconds **(B)** 3 seconds **(C)** 4 seconds
   (D) 5 seconds **(E)** 6 seconds

   (b) At what time does the stone reach its maximum height?

   (A) 1 second **(B)** 3/2 seconds **(C)** 2 seconds
   (D) 5/2 seconds **(E)** 3 seconds

3. Which of the following is a horizontal asymptote of $r(x)$? Circle all those that apply.

$$r(x) = \frac{(x+4)(x^2-1)(3x^2-4)}{(x^2+x-12)(x-1)^4}$$

   (A) $x = 1$ **(B)** $x = 3$ **(C)** $y = 0$ **(D)** $y = 1$ **(E)** $y = 3$

4. Referring again to the function $r(x)$ in the previous problem, which of the following is a vertical asymptote. Again circle all that apply.

   (A) $y = 0$ **(B)** $x = -4$ **(C)** $x = -1$ **(D)** $x = 1$ **(E)** $x = 3$

On all the following questions, **show your work.**

5. (10 points) Sketch the graph of a function f with domain $[-4, -2) \cup (-2, 4]$ on the coordinate axes provided that has all the following properties:

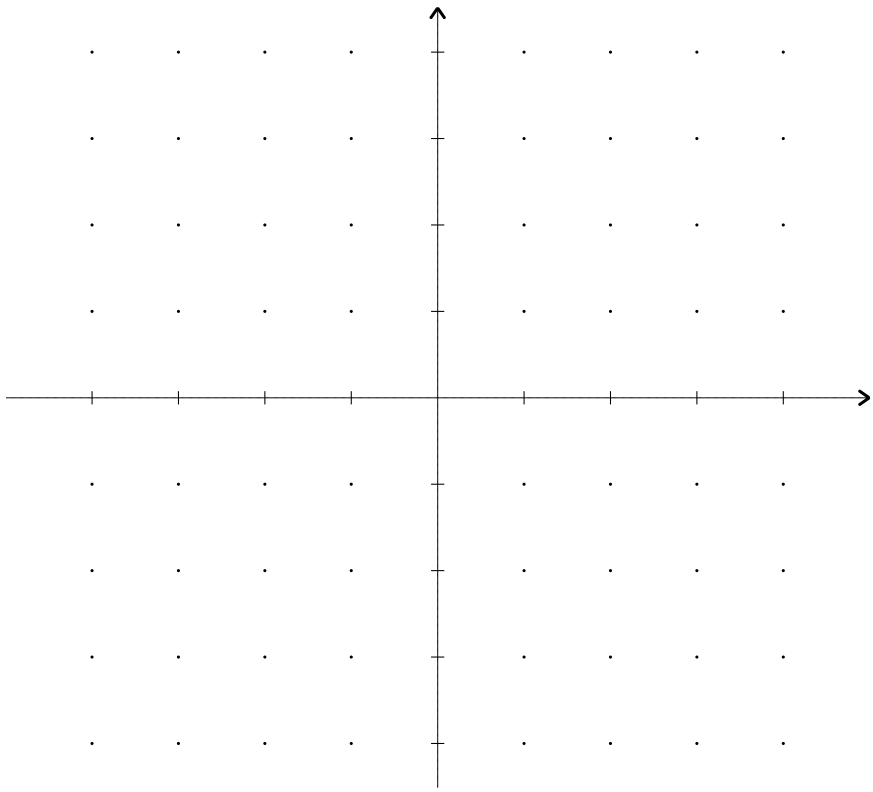
   (a) $f(-4) = f(-1) = 0$ and $f(0) = -1$.

   (b) f has a stationary point (ie, $f' = 0$) at $x = 1/2$.

   (c) f has a singular point (ie, f' is undefined) at $x = 3$.

   (d) f has a local minimum at $1/2$.

   (e) f has a vertical asymptote at $x = -2$.



6. (40 points) Let $g(x) = (2x-3)^3(x+1)^2$.

   (a) Find $g'(x)$ and the critical points of g. Express g' in factored form.

   (b) Find $g''(x)$, and express it in factored form. Use the second derivative test and other methods to classify the points found in part a as relative minimums, relative maximums, or neither.

   (c) Use the test interval technique to determine the intervals over which g is increasing.

   (d) Use the test interval technique to determine the intervals over which g is concave upwards

7. (20 points) A topless box is constructed from a rectangular piece of cardboard that measures 16 inches by 12 inches. An x by x square is cut from each of the four corners, and the sides are then folded upwards to build the box.

   (a) Express the volume V as a function of x.

   (b) Use the physical constraints to find the domain of V.

   (c) Find the derivative of V and use it to find the critical points of V.

   (d) Compute the value of V at all the points in the domain, including endpoints, where V could have an absolute extrema.

   (e) What is the volume of the largest box that can be so constructed?

8. (16 points) Optimal Charter Flight Fare. If exactly 160 people sign up for a charter flight, the agency charges \$300. However, if more than 160 sign up, the agency reduces the fare by \$0.80 for each additional person.

   (a) Let x denote the number of passengers beyond 160. Construct the revenue function $R(x)$.

   (b) Find all the critical points of your revenue function.

   (c) What number of passengers results in the maximum revenue?

   (d) What is the maximum revenue?