March 27, 2002 Your name ______________________

The first 7 problems count 5 points each. Problems 8 through 11 are multiple choice and count 7 points each and the final ones counts as marked. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 8 through 11, but you must show your work on the other problems. The total number of points available is 158.

Each of the next few items are true-false. To get full credit you must give a valid reason for your answer. Circle either True or False, and give your reason in the space provided. Generally, 2 points for the right t/f value and 3 points for the right reason.

1. True or false. If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.

2. True or false. If $f''(x) < 0$ on the interval (a, c) and $f''(x) > 0$ on the interval (c, b), then the point $(c, f(c))$ is a point of inflection of f.

3. True or false. If $f'(c) = 0$, then f has a relative maximum or a relative minimum at $x = c$.

4. True or false. If f has a relative maximum at $x = c$, then $f'(c) = 0$.

5. True or false. If $f'(c) = 0$ and $f''(c) < 0$, then f has a relative maximum at $x = c$.

6. True or false. If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.

7. True or false. If $h(x) = \sqrt{x^2 - 4}$, then $h'(x) = \frac{1}{2}(x^2 - 4)^{-1/2}$.

The next few problems are multiple choice. There is no need to justify your answers.

8. Over which of the intervals is the function $f(x) = x^4 - 4x^3 - 18x^2 + 2x + 1$ concave down?

   (A) $[-2, 0]$ **(B)** $[-1, 2]$ **(C)** $[1, 4]$ **(D)** $[2, 6]$ **(E)** $[3, 8]$

9. The line tangent to the graph of $g(x) = 3x^2 - 5x$ at the point $(1, -2)$ has a y-intercept of

   (A) -3 **(B)** -2 **(C)** -1 **(D)** 1 **(E)** 2

10. The absolute maximum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 4$ on the interval $-2 \leq x \leq 2$ is

    (A) -10 **(B)** 0 **(C)** 9 **(D)** 10 **(E)** 12

11. The absolute maximum value value of the function $f(x) = 2x^3 - 9x^2 + 12x + 4$ on the interval $-2 \leq x \leq 3$ is

    (A) -10 **(B)** 0 **(C)** 9 **(D)** 12 **(E)** 14

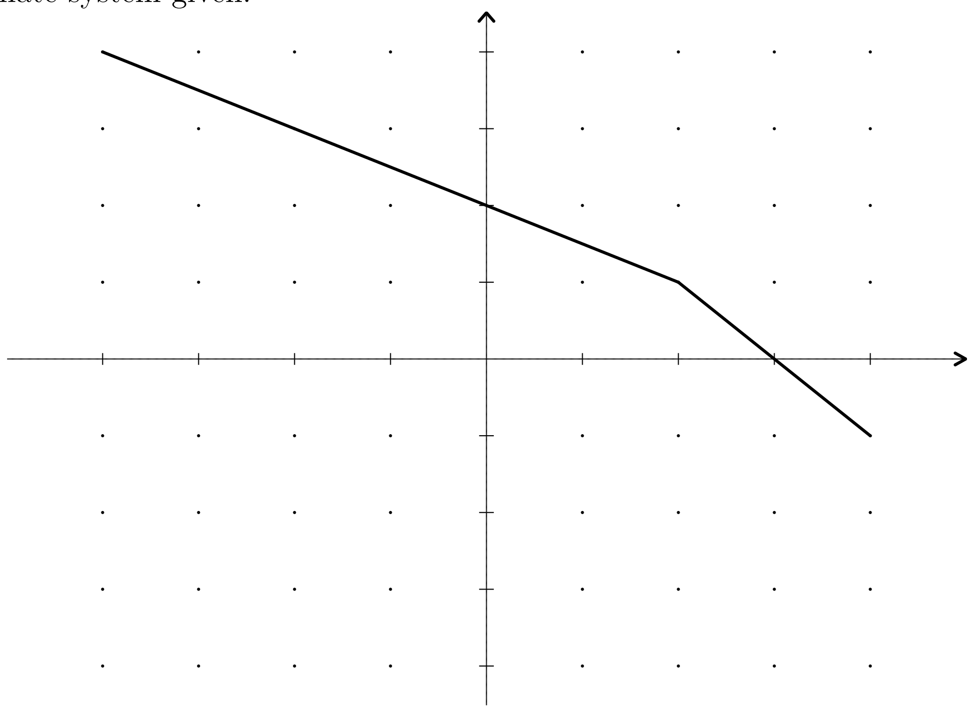
On all the following questions, **show your work.**

12. (12 points) Let $g(x) = (2x-6)^2(x+3)^2$.

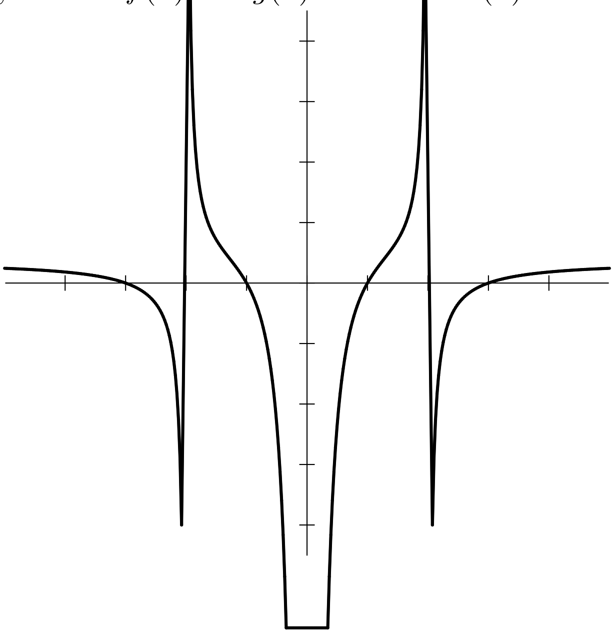
    (a) Use the test interval technique (not a graphing calculator) to find the intervals over which g is increasing.

    (b) Find and classify each critical point as a location of a. a relative maximum, b. a relative minimum, or c. neither a relative max nor a relative min.

13. (8 points) Sketch and example of a continuous function $f(x)$ that has a domain of $[-4, 4]$, and has singular point at $x = 2$ and a value of 1 at $x = 2$, but does not have a relative maximum or a relative minimum at $x = 2$. Use the coordinate system given.



14. (15 points) The function $H(x)$ shown below is a rational function with four zeros, $x = \pm 1$ and $x = \pm 3$, and vertical asymptotes at $x = -2$ and $x = 2$. Notice that H also has another vertical asymptote. It also has $y = 1/2$ as a horizontal asymptote. Find a symbolic representation of $H(x)$. In other words, find two polynomials $f(x)$ and $g(x)$ such that $H(x)$ could be $f(x)/g(x)$.



15. (15 points) Consider the rational function

$$f(x) = \frac{(x^2-4)(x+1)}{(2x^2-3)(x-2)}.$$

   (a) Find the horizontal asymptote(s).

   (b) Find the vertical asymptotes.

   (c) Compute $\lim_{x\to\infty} f(x)$.

16. (15 points) The quantity demanded per month, x of a certain brand of electric shavers is related to the price, p, per shaver by the equation $p = -0.2x + 1000 \quad (0 < x < 20,000)$, where p is measured in dollars. The total monthly cost for manufacturing the shavers is given by $C(x) = 0.0001x^3 - 0.3x^2 + 1000x + 2000$. Recall that the revenue is the product of the demand and the price per unit. Construct the revenue function, $R(x)$. How is the profit related to revenue and cost? Find $P'(x)$, where $P(x)$ denotes the profit function. How many shavers should be produced per month in order to maximize the company's profit? What is the maximum profit?

This part of the test is for you to take home. Its due next class meeting. You're on your honor not to discuss it with anyone. You must sigh a pledge that you have not done so. Sign below: I have neither given nor received help on either of these problems from anyone.

____

For both problems below, let F be the number of letters in your first (= given) name, and let L be the number of letters in your last (= family) name. The point is to customize the problems for you.

17. (15 points) Four congruent $x \times x$ squares from the corners of a cardboard rectangle that measures $2F \times 2L$. The sides are then folded upward to form a topless box. Find the volume V as a function of x. What is the logical domain? Compute $V(0), V(1), V(2)$, and $V(3)$. Find $V'(x)$ and use this to determine the critical points of V. Find the absolute maximum value of V and the value of x where it occurs.

18. (15 points) You're on an island at the point D that is F miles from the closest on shore point A. The point A is $2L$ miles from the point B where you need to get as soon as possible. You can swim at 2 miles per hour and you can run at 6 miles per hour. You decide to swim to a point C between A and B that is x miles from A, then run from C to B. What is the distance from A to C as a function of x? What is the distance from C to B as a function of x? Find the time it takes to make the swim-run journey if $x = 0$ miles, $x = 1$ mile, and $x = 2$ miles. Then find the time $T(x)$ required to make the trip when x is any reasonable value. What is the domain of T? What are the critical points of T? What is the absolute minimum value of $T(x)$?