November 30, 2016 **Name** ______________________________

The problems count as marked. The total number of points available is 146. Throughout this test, **show your work.**

1. (10 points) How long does it take an investment of $\$P$ at an annual rate of 8% to triple in value if compounding

   (a) takes place quarterly? Round your answer to the nearest tenth of a year.

   (b) takes place continuously? Round your answer to the nearest tenth of a year. As usual, no work shown, no credit!

2. (12 points) Let $f(x) = e^{x^3-27x}$

   (a) Find an equation for the line tangent to $f(x)$ at the point $(0, f(0))$.

   (b) Find an interval over which $f(x)$ is decreasing.

3. (20 points) Consider the function $f(x) = \ln(x^3 - x + 1)^{1/2}$.

   (a) Find $f'(x)$.

   (b) Find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

   (c) Find $f''(x)$.

   (d) Find the sign chart for $f''(x)$.

   (e) Find the intervals over which f is concave upwards.

4. (30 points) Let $H(x) = (x^2 - 9)^3(3x - 1)^2$.

   (a) Use the chain and product rules to find $H'(x)$.

   (b) Find the critical points of H.

   (c) Build the sign chart for $H'(x)$

   (d) Classify the critical points of H as max, min, or imposters.

   (e) Find the intervals over which H is increasing.

5. (12 points) Polonium is a radioactive element which decays according to the law
$$Q(t) = Q_0 \cdot 2^{-t/140}.$$
where Q_0 is the initial amount and t is measured in days.

   (a) If the amount of material left after 280 days is 20 mg, what was the initial amount?

   (b) What is half-life of Polonium?

   (c) How much Polonium will be left after 400 days?

6. (15 points) Living with HIV. On the basis of data from WHO, the number of people living with HIV worldwide from 1985 through 2006 is given by the logistic curve

$$N(t) = \frac{40}{1 + 19e^{-0.30t}}.$$

where $N(t)$ is measured in millions and t is measured in years, with $t = 0$ corresponding to the beginning of 1985.

(a) How many people were living with HIV at the beginning of 1985?

(b) Assuming the trend continues, how many people living with HIV would we expect at the beginning of 2010?

(c) At what rate is the disease growing at the beginning of the year 2000? Be sure to include your units in the answer.

7. (20 points) Demand for IPhones. Apple economists propose that the monthly demand for IPhone 7's is modeled by

$$D(t) = 20 - 15e^{-kt},$$

where t is measured in months and $D(t)$ is measured in millions of phones. Suppose that 6 million phones are sold during the third month of the year ($t = 3$).

   (a) How many phones can Apple expect to sell during the fourth month of the year?

   (b) What is the demand after a year?

   (c) At what level of sales is demand expected to stabilize?

   (d) Explain why this model does not work for items like IPhone 7's.

8. (15 points) Let $f(x) = \ln[(x^2+1)(x^3+x^2)(\frac{1}{x+2})]$.

   (a) Find the derivative of $f(x)$.

   (b) What is $f'(1)$?

   (c) Find an equation for the line tangent to f at the point $(1, f(1))$. Leave your answer in terms if the ln function.

9. (20 points) A botanist conjectures that the height of a certain type of pine tree can be modeled by a learning curve. To test his conjecture, he plants a 2 foot tall tree. He knows that eventually the tree will grow to 40 feet tall, its maximum height. Suppose that after one year, the tree is 4 feet tall.

   (a) What does the model predict for the height of the tree after two years.

   (b) How many inches does the tree grow during the fourth year?

   (c) What is the instantaneous rate of growth at $t = 3.5$ years.

   (d) Describe the connection between the two answers (b) and (c).