**November 29, 2017** **Name** ______________________________

The total number of points available is 163. Throughout this test, **SHOW YOUR WORK**. Throughout this test, you are expected to use calculus to solve problems. Graphing calculator solutions will generally be worth substantially less credit.

1. (12 points) Find an equation for the line tangent to the graph of $f(x) = xe^{-2x+4}$ at the point $(2, f(2))$.

2. (12 points) In 1985, the tuition at Yale University was \$10000 per year. In 2015 it was about \$44000 per year. Estimate the annual percent growth. Write a sentence to justify your answer.

3. (12 points) Find an equation for the line tangent to the graph of $f(x) = x^2 \ln(x)$ at the point $(1, f(1))$.

4. (40 points) There is a function $g$ whose derivative is given below:

$$g'(x) = \begin{cases} x + 17 & \text{if } -10 \le x \le -3 \\ x^2 - 3x - 4 & \text{if } -3 < x \le 6 \\ 14 & \text{if } 6 < x \le 10 \end{cases}$$

(a) What is the domain of $g'$. Use interval notation.

(b) Find the critical points of $g'$.

(c) Find the intervals over which $g'$ is increasing.

(d) Find the intervals over which the function $g$ is decreasing.

(e) Find the critical points of $g$.

(f) Find the absolute maximum and absolute minimum of $g'$. You must show all your work.

5. (42 points) Exponentials and Logarithms.

   (a) You invest \$1000 at 8% for three years compounded quarterly. How much more would the final amount be if the investment is compounded continuously.

   (b) Suppose the half life of a radioactive substance is one year. Find the decay constant $k$.

   (c) Suppose the time required for a continuously compounded investment to triple is 12 years. What is the time required to double.

   (d) Suppose $Q(t) = \frac{A}{1+Be^{-kt}}$ is used to model the growth of a rumor in a population of 1000 where $t$ is measured in hours. The rumor starts at a party attended by 100 people. After one hour, 200 people have heard the rumor. What is $k$?

   (e) Solve the equation $\ln(x+2) - \ln(4x+3) = \ln(1/x)$.

   (f) Solve for $x$: $2e^{2x} - 11e^{x} + 15 = 0$.

6. (20 points) Consider the function $f(x) = \frac{(2x^2+4)(x-3)}{x(x-1)}$. Follow the steps below to build the line tangent to $f$ at the point $(4, f(4))$. This procedure is called logarithmic differentiation.

   (a) Let $G(x) = \ln(f(x))$. Find $G'(x)$.

   (b) Note that $G'(x) = \frac{f'(x)}{f(x)}$. What is $f(4)$?

   (c) Note that $G'(4) = f'(4)/f(4)$. Use this fact to find $f'(4)$.

   (d) Build the line tangent to $f$ at $(4, f(4))$

7. (25 points) Consider the function $f(x) = \ln(3x^2 + 1)$.

   (a) Find $f'(x)$.

   (b) Find an equation for the line tangent to the graph of $f$ at the point $(3, f(3))$.

   (c) Find $f''(x)$.

   (d) Find the sign chart for $f''(x)$.

   (e) Find the intervals over which $f$ is concave upwards.