

April 10, 2019

Name _____

The problems count as marked. The total number of points available is xxx.

Throughout this test, **SHOW YOUR WORK.**

1. (15 points) Consider the cubic curve $f(x) = 2x^3 + 3x^2 - 36x + 17$.
 - (a) Build the sign chart for $f'(x)$.
 - (b) Using the sign chart for $f'(x)$, find the intervals over which $f(x)$ is increasing. .
 - (c) Find a point of inflection on the graph of f .

2. (15 points) Consider the cubic curve $g(x) = e^{x^3-12x}$.
 - (a) Find $g'(x)$ and $g''(x)$.
 - (b) Build the sign chart for $g''(x)$.
 - (c) Use the information in (b) to discuss the concavity of g . No points for a bold answer without reference to the sign chart.

3. (30 points) Find the critical points for each of the functions given below. For credit, you must show the equation you're solving to get the critical points.
 - (a) $f(x) = (x - 3)^{\frac{2}{3}}$.
 - (b) $g(x) = \ln(x^3 - 3x + 22)$.
 - (c) $h(x) = \left(\frac{2x-1}{3x+1}\right)^4$
 - (d) $f(x) = e^{2x} - 5x$
 - (e) $k(x) = \ln(6x^2 + 5x + 2) - x$.

4. (15 points) Meliha invests \$1000 at a rate of r percent compounded continuously. After 16 years her investment is worth \$4000.

(a) How long did it take for her \$1000 investment to double?

(b) How long did it take her investment to triple?

5. (15 points) Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after t weeks is given by

$$F(t) = 120 - 40e^{-.4t}$$

(a) How many weeks into the course does it take for Rachel to reach a speed of 100 words per minute.

(b) During the third week of the class, at what rate is Rachel's typing speed increasing?

6. (10 points) The population of the world in 1990 was 5 billion and the relative growth rate was estimated at 1.5 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2010.
7. (10 points) Let $g(x) = x \ln(x)$. Notice that $g(e) = e \ln(e) = e$. Find an equation for the line tangent to g at the point (e, e) .
8. (20 points) Consider the function $f(x) = \ln(x^2 + 1)$.
- (a) Find $f'(x)$.
- (b) Find $f''(x)$.
- (c) Find the sign chart for $f''(x)$.
- (d) Find the intervals over which f is concave upwards.