

April 23, 2004

Name _____

The total number of points available is 120. Throughout this test, **show your work.**

1. (15 points) How long does it take an 8% investment to triple if

(a) Compounding takes place monthly?

(b) Compounding takes place continuously?

2. (15 points) Let $f(x) = x^4 + 2x^3 - 12x^2 + x - 5$.

(a) Find the interval(s) where f is concave upward.

(b) Find the inflection points of f , if there are any.

3. (15 points) Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3 - 6x^2 + 8x + 7$ on the interval $0 \leq x \leq 6$.

4. (15 points) Suppose the function $Q(t) = Q_0 e^{-kt}$ satisfies $Q(5770) = Q_0/2$.

(a) What is the value of k ?

(b) For what value of t is it true that $Q(t) = Q_0/4$?

(c) Find $Q'(t)$.

(d) What is the rate of growth of $Q(t)$ at $t = 11540$?

5. (15 points) Find the interval(s) where $f(x) = (x - 4)(x^2 - 1)(x + 3)$ is positive.

6. (15 points) Compute each of the following derivatives.

(a) $\frac{d}{dx} \sqrt{x^3 + 1}$

(b) $\frac{d}{dx} \ln(x^3 + 1)$

(c) $\frac{d}{dx} \frac{e^x}{x}$

(d) $\frac{d}{dx} (x - 3)^3 (x^2 + 1)^4 (x - 8)^{12}$

7. (15 points) The quantity demanded each month of the Sicard wristwatch is related to the price by the equation

$$p = \frac{50}{0.01x^2 + 1}$$

for $0 \leq x \leq 20$ where p is measured in dollars and x is measured in units of a thousand.

- (a) Find the demand when the price is set at \$25 per watch.
- (b) Recall the revenue function is the product of the price and the number of units sold. Find the revenue function $R(x)$.
- (c) Use the results of part b. to find the number of (thousands of) units needed to maximize the revenue.

8. (15 points) Four identical $x \times x$ square corners are cut from a 14×18 inch rectangular piece of metal, and the sides are folded upward to build a box.
- (a) What is the volume of the box that results when the corners cut are 1×1 .
- (b) Let $V(x)$ denote the volume of the box when the $x \times x$ corners are removed. Find $V(2)$ and $V(3)$.
- (c) What is the implied domain of V ?
- (d) Find $V'(x)$.
- (e) Find the critical points of $V(x)$.
- (f) What value of x makes the value of V maximum? Estimate within .01 the maximum value of V .