

October 29, 2004

Name _____

The total number of points available is 135. Throughout this test, **show your work.**

1. (25 points) Let $g(x) = x^3 - 6x^2 - 15x + 32$.

(a) Find the critical points of g .

(b) Find the intervals over which g is increasing.

(c) Find the intervals over which g is concave upward.

(d) Find the locations of local maxima and minima for g .

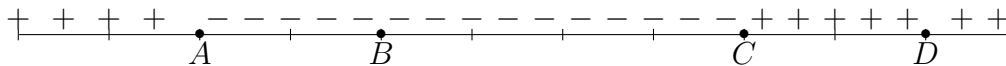
(e) What is the maximum value of g over the interval $[0, 10]$?

2. (20 points) Find the critical points of each function.

(a) $f(x) = (x - 4)^2(2x - 3)^3$

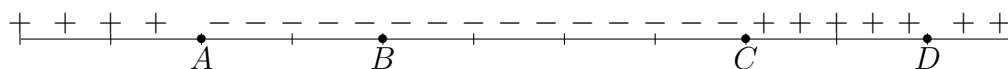
(b) $g(x) = (x^2 - 4)^{2/3}$

3. (20 points) Given below is a sign chart for the derivative $f'(x)$ of a function.

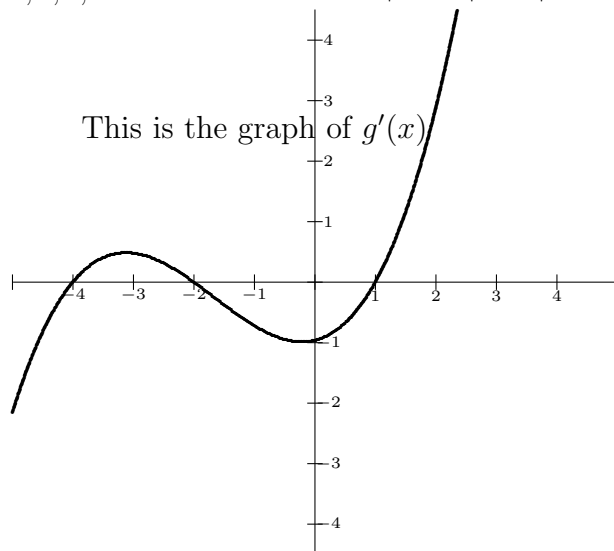


(a) For each of the stationary points A, B, C and D tell whether $f(x)$ has a relative maximum, relative minimum, or neither at the point.

(b) Suppose $f(x)$ is a polynomial function. Sketch a function on the coordinate system below that could have a derivative whose sign chart is the one given.



4. (15 points) Consider the cubic polynomial $p(x)$ whose graph is given. Note the x -intercepts are -4 , -2 and 1 and the y -intercept is -1 . Find numbers a , b , c , and d such that $ax^3 + bx^2 + cx + d$ has the given graph.



5. (20 points) The altitude of a rocket in feet t seconds into the flight is given by $s = f(t) = -t^3 + 96t^2 + 195t + 5$.

(a) What is the maximum altitude attained by the rocket? At what time into the flight does this occur?

(b) What is the maximum velocity attained by the rocket? At what time into the flight does this occur?

6. (15 points) Four identical $x \times x$ square corners are cut from a 14×20 inch rectangular piece of metal, and the sides are folded upward to build a box.
- (a) What is the volume of the box that results when the corners cut are 1×1 .
- (b) Let $V(x)$ denote the volume of the box when the $x \times x$ corners are removed. Find $V(2)$ and $V(3)$.
- (c) What is the implied domain of V ?
- (d) Find $V'(x)$.
- (e) Find the critical points of $V(x)$.
- (f) What value of x makes the value of V maximum? Estimate within .01 the maximum value of V .

7. (20 points) Amber Airlines runs chartered flights to Costa Rica. They expect 200 passengers and they charge each passenger \$300. However if more than 200 persons sign up for the flight, they agree to charge \$0.75 less per ticket for each extra person.
- (a) Find the revenue function $R(x)$ in terms of the number of new passengers x . In other words, let $x + 200$ represent the number of passengers, where $x > 0$

(b) How many passengers result in the maximum revenue?

(c) What is that maximum revenue?