

April 10, 2007

Name \_\_\_\_\_

The total number of points available is 139. Throughout this test, **show your work.**

1. (10 points) Consider the function  $f(x) = \ln(x^2 + 1)$ .
  - (a) Find the function  $f'(x)$ .
  
  
  
  
  
  
  
  
  
  
  - (b) Find  $f'(2)$  and use this number to find an equation for the line tangent to the graph of  $f$  at the point  $(2, \ln 5)$ .
  
  
  
  
  
  
  
  
  
  
2. (12 points) Consider the function  $f(x) = x^3 - 5.5x^2 - 4x + 7$ ,  $-4 \leq x \leq 6$ . Find the locations of the absolute maximum of  $f(x)$  and the absolute minimum of  $f(x)$  and the value of  $f$  at these points.

3. (30 points including 10 for part (e)) For each function listed below, find all the critical points. Tell whether each critical point gives rise to a local maximum, a local minimum, or neither.

(a)  $f(x) = (x^3 - 8)^2$

(b)  $h(x) = e^{2x}/x$

(c)  $h(x) = x^2 \ln(x + 2)$

(d)  $g(x) = (x - 1)^{2/3}$

(e)  $k(x) = (2x - 3)^2(x^2 - 1)$

4. (15 points) The function  $f(x)$  has been differentiated twice to get  $f''(x) = (x^2 - 4)(x - 5)(x + 1)$ .

(a) Find the places where  $f''(x)$  changes signs.

(b) Find the places where  $f'(x)$  has a horizontal tangent line.

(c) Find the intervals over which  $f(x)$  is concave upwards.

5. (15 points) Note to the class. There was a typo (% instead of \$) on the test, so the solution to the problem as stated is quite different from the one given here. When a management training company prices its seminar on management techniques at \$400 per person, 1000 people will attend the seminar. The company estimates that for each \$5 reduction in the price, an additional 20 will attend the seminar. How much should the company charge for the seminar in order to maximize its revenue? What is the maximum revenue?

6. (15 points) It is known from past experiments that the height (in feet) after  $t$  months is given by

$$H(t) = 4t^{1/2} - 2t, \quad 0 \leq t \leq 2.$$

How long, on average, does it take a plant to reach its maximum height? What is the maximum height?

7. (15 points) For a particular person learning to type, it was found that the number  $N$  of words per minute the person was able to type after  $t$  hours of practice, was given by

$$N = N(t) = 100(1 - e^{-0.02t}).$$

- (a) After 10 hours of practice how many words per minute could the person type?
- (b) What was the rate of improvement after 10 hours of practice?
- (c) What was the rate of improvement after 40 hours of practice?