Math 1120, Section 7 **Calculus** **Test 3**

November 20, 2007 **Name** ________________________________

The total number of points available is 134. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = 1+9x+3x^2-x^3, \quad -2 \leq x \leq 6$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

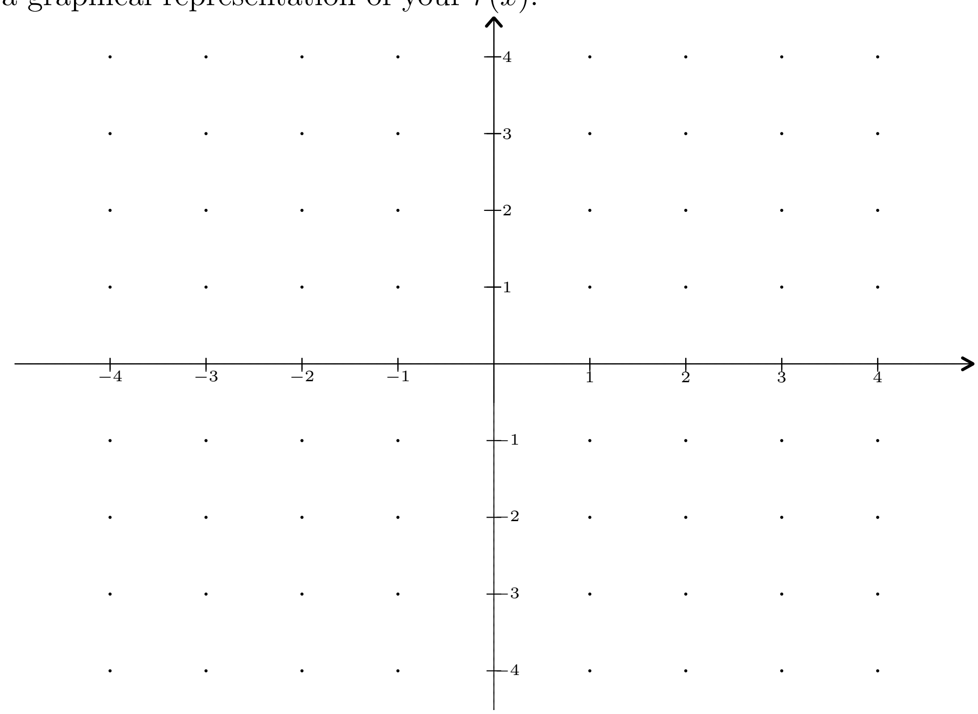
2. (25 points) Find a symbolic rational function $r(x)$ that has all the following properties:

   i. It has exactly two zeros, $x = -2$ and $x = 3$.

   ii. It has two vertical asymptotes, $x = 0$ and $x = -3$.

   iii. It has $y = 2$ as a horizontal asymptote.

   (b) Find a graphical representation of your $r(x)$.



   (a) Find a symbolic representation of r.

3. (15 points) A rancher wants to fence in an area of 20 square miles in a rectangular field and then divide it into three smaller rectangular fields using two segments of fence parallel to one side. What is the shortest length of fence that the rancher can use?

4. (25 points) Consider the function $f(x) = \ln(3x^2 + 1)$.

   (a) Find $f'(x)$.

   (b) Find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

   (c) Find $f''(x)$.

   (d) Find the sign chart for $f''(x)$.

   (e) Find the intervals over which f is concave upwards.

5. (10 points) Find the point on the line $y = 3$ that is closest to the point $(4, -3)$. What is that shortest distance?

6. (20 points) Four identical $x \times x$ square corners are cut from a 10×10 inch rectangular piece of metal, and the sides are folded upward to build a box.

   (a) What is the volume of the box that results when the corners cut are 1×1.

   (b) Let $V(x)$ denote the volume of the box when the $x \times x$ corners are removed. Find $V(2)$ and $V(3)$.

   (c) What is the implied domain of V?

   (d) Find $V'(x)$.

   (e) Find the critical points of $V(x)$.

   (f) What value of x makes the value of V maximum?

7. (12 points) Compound Interest.

   (a) Consider the equation $2000(1+0.03)^{4t} = 6000$. Find the value of t and interpret your answer in the language of compound interest.

   (b) Consider the equation $P(1+0.04)^{4\cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

   (c) Consider the equation $Pe^{10r} = 2P$. Solve for r and interpret your answer in the language of compound interest.

8. (12 points) Find an equation for the line tangent to the graph of $f(x) = xe^{-2x+4}$ at the point $(2, f(2))$.