November 24, 2008 **Name** ______________________________

The total number of points available is 137. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = (x^2 - 4x + 4)e^{2x}$.

   (a) Use the product rule to find $f'(x)$.

   (b) List the critical points of f.

   (c) Construct the sign chart for $f'(x)$.

   (d) Write in interval notation the interval(s) over which f is increasing.

2. (15 points) Consider the function $f(x) = \ln[(2x^2 + 3)(7x - 2)(x^2 - 4)]$.

   (a) Recalling that $\ln(x)$ is defined precisely when $x > 0$, find the domain of f.

   (b) Let $g(x) = (2x^2 + 3)(7x - 2)^2(x^2 - 4)^3$. Use logarithmic differentiation to find g'. You need not simplify your answer.

3. (15 points) Consider the function $f(x) = \ln(x^2 + 4)$.

   (a) Find the domain of f.

   (b) Find both $f'(x)$ and $f''(x)$.

   (c) Build the sign chart for $f''(x)$.

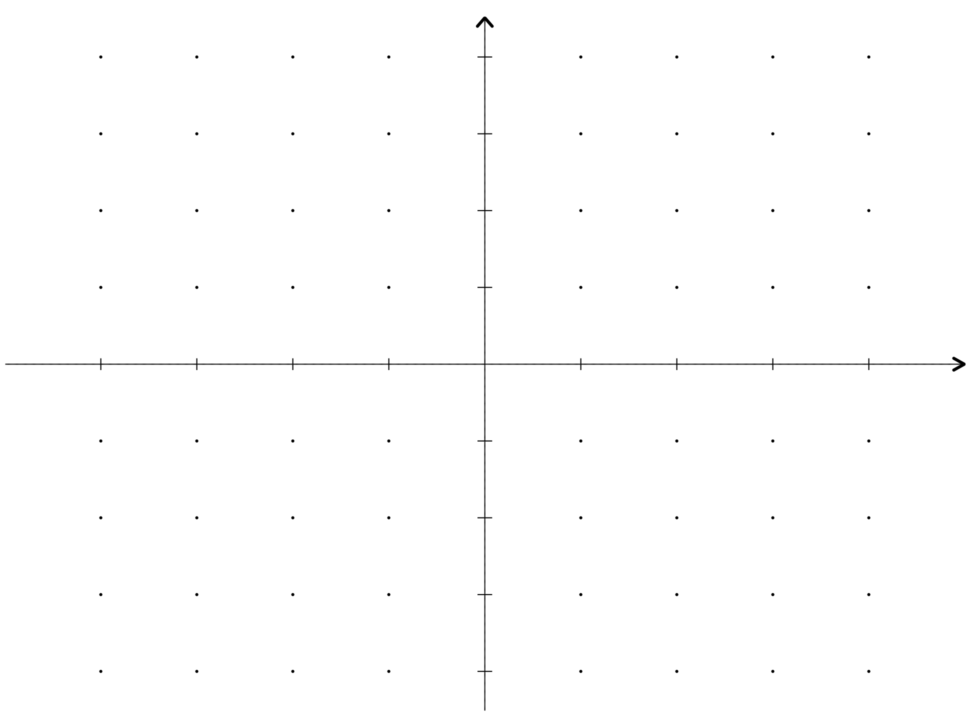
   (d) Does f have an absolute maximum or and absolute minimum? Is so, which? ... and where?

   (e) Find an interval over which f is increasing. As usual, to get credit for this, you must show your work.

   (f) Does f have any inflection points?

   (g) Use the information from part b. to find the intervals where f is concave upwards.

   (h) Sketch the graph of f.



4. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $68°F$, A and k are constants, and t is expressed in minutes.

   (a) What is the value of A?

   (b) Suppose that after exactly 8 minutes, the temperature of the coffee is $136.6°F$. What is the value of k?

   (c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

5. (10 points) Find an equation for the line tangent to the graph of $y = \ln x^2$ at the point $(1, 0)$.

6. (10 points) Find an equation for the line tangent to the graph of $y = e^{5x-2}$ at the point $(2/5, 1)$.

7. (12 points) A skull from an archeological dig has one-twelfth the amount of Carbon-14 it had when the specimen was alive.

   (a) Recall that the half-life of Carbon-14 is 5770 years. Find the decay constant k.

   (b) What is the age of the specimen? Round off your answer to the nearest multiple of one hundred years.

8. (10 points) How long does it take an investment of $\$P$ at an annual rate of 8% to triple in value if compounding

   (a) takes place quarterly? Round your answer to the nearest tenth of a year.

   (b) takes place continuously? Round your answer to the nearest tenth of a year. As usual, now work shown, no credit!

9. (30 points) Spreading of a rumor. Three hundred college students attend a lecture of the Dean at which she hints that the college will become coed. The rumor spreads according to the logistic curve

$$Q(t) = \frac{3000}{1 + Be^{-kt}},$$

where t is measured in hours.

(a) Compute the parameter B.

(b) How many students attend the college? Hint: the question is not 'how many attended the lecture?'

(c) Two hours after the speech, 600 students had heard the rumor. How many students had heard the rumor after four hours?

(d) How fast is the rumor spreading after four hours?

(e) After how many hours will 2000 students have heard the rumor?