April 9, 2009 **Name** ______________________________

The problems count as marked. The total number of points available is 160. Throughout this test, **show your work.**

1. (15 points) Consider the cubic curve $f(x) = 2x^3 + 3x^2 - 36x + 17$.

   (a) Build the sign chart for $f'(x)$.

   Solution: $f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x+3)(x-2)$, which is negative over $(-3, 2)$ and positive elsewhere.

   (b) Using the sign chart for $f'(x)$, find the intervals over which $f(x)$ is increasing. .

   Solution: Since $f'(x)$ is positive on $(-\infty, -3)$ and on $(2, \infty)$, f is increasing over those intervals.

   (c) Find a point of inflection on the graph of f.

   Solution: $f''(x) = 12x + 6 = 6(2x+1)$, which changes signs at $x = -1/2$, so there is a point of inflection at $(-1/2, f(-1/2)) = (-0.5, 35.5)$.

2. (15 points) Consider the cubic curve $g(x) = e^{x^2 - x}$.

   (a) Find $g'(x)$ and $g''(x)$.

   Solution: $g'(x) = e^{x^2-x}(2x-1)$ and $g''(x) = 2e^{x^2-x} + e^{x^2-x}(2x-1)^2 = e^{x^2-x}(4x^2 - 4x + 3)$.

   (b) Build the sign chart for $g''(x)$.

   Solution: Since the discriminant $D = b^2 - 4ac$ of $4x^2 - 4x + 3$ is negative, it follows that $g''(x) > 0$ for all x.

   (c) Use the information in (b) to discuss the concavity of g. No points for a bold answer without reference to the sign chart.

   Solution: Since $g''(x) > 0$ for all x, g must be concave upwards over $(-\infty, \infty)$.

3. (30 points) Let $h(x) = \frac{(2x+7)(2x+3)}{(x-1)(2x-5)}$.

   (a) Find the asymptotes and the zeros of h.

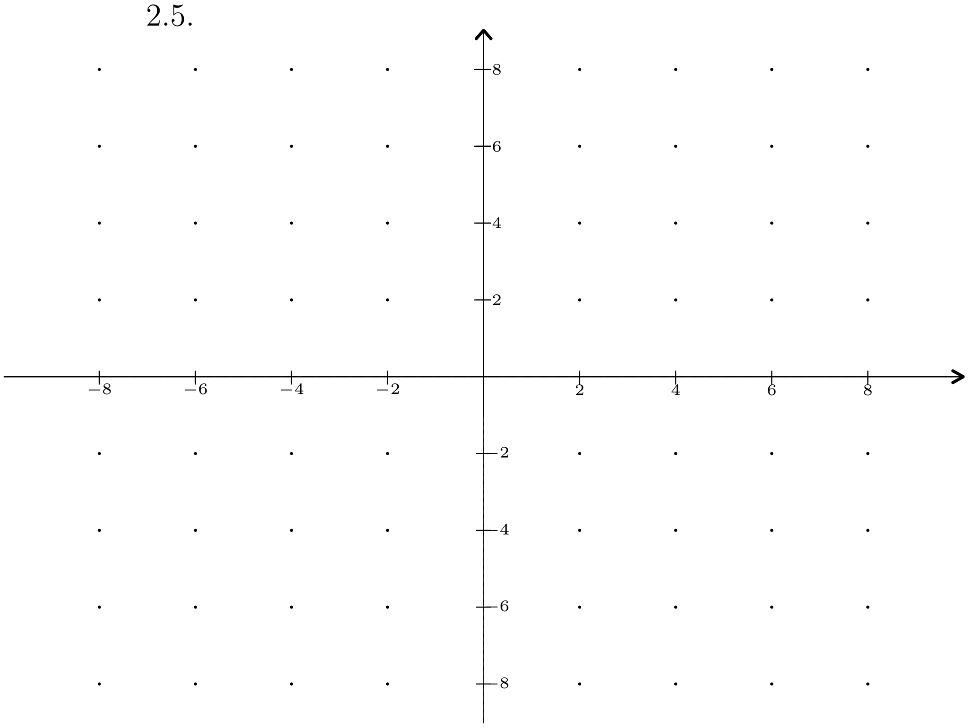
   Solution: Solve $2x + 7 = 0$ to get $x = -7/2$ and solve $2x + 3 = 0$ to get $x = -3/2$ for zeros, and $x = 1, x = 5/2$, and $y = 2$ for asymptotes.

   (b) Build the sign chart for $h(x)$.

   Solution: The sign chart shows that h is positive over $(-\infty, -7/2)$, $(-3/2, 1)$ and $(5/2, \infty)$, and negative on the open intervals $(-7/2, -3/2)$ and $(1, 5/2)$.

   (c) Sketch the graph of $h(x)$ USING the information in (a) and (b).

   Solution: The graph must show that there are relative extrema at two values, a minimum between -2 and -3 and a maximum between 1 and 2.5.



4. (30 points) Find the critical points for each of the functions given below. For credit, you must show the equation you're solving to get the critical points.

   (a) $f(x) = (x-3)^{\frac{2}{3}}$.
   Solution: $f'(x) = \frac{2}{3}(x-3)^{-1/3}$, so $x = 3$ is a singular point.

   (b) $g(x) = \ln(x^3 - 3x + 22)$.
   Solution: $g'(x) = \frac{3x^2-3}{x^3-3x+22}$ so $x = \pm 1$ are stationary points.

   (c) $h(x) = \left(\frac{2x-1}{3x+1}\right)^4$
   Solution: $h'(x) = 4\left(\frac{2x-1}{3x+1}\right)^3 \cdot \frac{2(3x+1)-3(2x-1)}{(3x+1)^2}$. This function has just one zero, at $x = -1/2$, and no singular points because h is not defined at $x = -1/3$.

   (d) $f(x) = e^{2x} - 5x$
   Solution: $f'(x) = 2e^{2x} - 5$ has just one zero, $x = \frac{\ln 5 - \ln 2}{2} \approx 0.458$

   (e) $k(x) = \ln(6x^2 + 5x + 2) - x$.
   Solution: $k'(x) = \frac{12x+5}{6x^2+5x+2} - 1$. Setting this equal to zero yields $12x+5 = 6x^2 + 5x + 2$, which is equivalent to $6x^2 - 7x - 3 = (3x+1)(2x-3)0$, so the critical points are $x = -1/$ and $x = 3/2$.

5. (15 points) Meliha invests \$1000 at a rate of r percent compounded continuously. After 16 years her investment is worth \$4000.

   (a) How long did it take for her \$1000 investment to double?

   Solution: The doubling time must be 8 years since the investment doubles twice in 16 years.

   (b) How long did it take her investment to triple?

   Solution: Solve $2P = Pe^{rt}$ for r when $t = 8$, so get $8r = \ln 2$ or $r = \ln 2/8 \approx 8.66\%$. Then solve $3P = Pe^{rt}$ for t when $r = \ln 2/8$, so get $t = \ln 3^8 \div \ln 2 \approx 12.68$ years.

6. (15 points) Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after t weeks is given by

$$F(t) = 120 - 40e^{-.4t}$$

   (a) How many weeks into the course does it take for Rachel to reach a speed of 100 words per minute.

   Solution: Solve $120 - 40e^{-.4t} = 100$ to get $t = 1.73$ weeks.

   (b) During the third week of the class, at what rate is Rachel's typing speed increasing?

   Solution: $F'(t) = 16e^{-0.4t}$, so $F'(3) = 16e^{-1.2} \approx 4.8$ words per minute.

7. (10 points) The population of the world in 1990 was 5 billion and the relative growth rate was estimated at 1.5 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2010.

   Solution: The function is $P(t) = p_0 e^{rt}$, and we know $P(0) = 5$ billion, so $P(20) = 5 \cdot e^{0.15(20)} = 5e^{0.3} \approx 6.749$ billion.

8. (10 points) Let $g(x) = x \ln(x)$. Notice that $g(e) = e \ln(e) = e$. Find an equation for the line tangent to g at the point (e, e).

   Solution: First note that, by the product rule, $g'(x) = \ln(x) + 1$. So $g'(e) = \ln(e) + 1 = 2$. Hence the line is $y - e = 2(x - e)$ which is $y = 2x - e$.

9. (20 points) Let P be the point $(2, 3)$ in the plane.

   (a) Find the point on the line $y = 4$ that is closest to P.

   Solution: Since $y = 4$ is a horizontal line, the closest point on it is the one directly above $(2, 3)$. That is, $(2, 4)$.

   (b) Find the point on the line $x = 4$ that is closest to P.

   Solution: Since $x = 4$ is a vertical line, the closest point on it is the one directly to the right of $(2, 3)$. That is, $(4, 3)$.

   (c) Find the point on the line $y = x+5$ that is closest to P. To get any credit for this part, you must show what equation you're solving and show how you solved it.

   Solution: The slope of the line $y = x + 5$ is 1 so the line with slope -1 through the point $(2, 3)$ goes through $y = x + 5$ at the point we want. That line is $y - 3 = -(x - 2)$ or $y = -x + 5$. The point they have in common is $(0, 5)$.