November 24, 2008 **Name** ______________________________

The total number of points available is 145. Throughout this test, **show your work.**

1. (10 points) Find an equation for the line tangent to the graph of $y = \ln(x^2 + 1)$ at the point $(1, \ln(2))$.

2. (10 points) Find an equation for the line tangent to the graph of $y = e^{2x-3}$ at the point $(3/2, 1)$.

3. (15 points) Let $H(x) = \ln(4x^2 + 12x + 10) - 2x$. Find all the critical points.

4. (20 points) Consider the function $f(x) = (2x - 4)e^{x^2}$.

   (a) Use the product rule to find $f'(x)$.

   (b) List the critical points of f.

   (c) Construct the sign chart for $f'(x)$.

   (d) Write in interval notation the interval(s) over which f is increasing.

5. (15 points) Consider the function $f(x) = \ln[(2x^2 + 3)(7x - 2)(x^2 - 4)]$.

   (a) Recalling that $\ln(x)$ is defined precisely when $x > 0$, find the domain of f.

   (b) Let $g(x) = (2x^2 + 3)(7x - 2)^2(x^2 - 4)^3$. Use logarithmic differentiation to find g'. Find a decimal representation of $g'(1)$.

6. (15 points) Sketch the graph of a function defined on the interval $[-4, 4]$ with the following properties.

   - It has zeros at $x = -2, x = 0$ and $x = 3$,
   - it has horizontal tangent lines at $x = -1$ and $x = 2$, and
   - it has a relative maximum at $x = -1$, a relative minimum at $x = 2$ and an absolute maximum value of 3 at the point $x = 4$.

7. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

   where T is the air temperature, $68°F$, A and k are positive constants, and t is expressed in minutes.

   (a) What is the value of A?

   (b) Suppose that after exactly 8 minutes, the temperature of the coffee is $136.6°F$. What is the value of k?

   (c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

8. (30 points) Exactly 50 rabbits were introduced to a tiny South Pacific island. After one year, there were 400 rabbits and after 20 years, there were 500 rabbits. Assuming logistic growth, find the three unknown parameters and answer the questions below. You may assume $\lim_{t\to\infty} Q(t) = Q(20)$.

   (a) What is the number of rabbits after 6 months?

   (b) What is the number of rabbits after 2 years?

   (c) At what time is the rate of growth of the rabbit population the greatest?

9. (10 points) How long does it take an investment of $\$P$ at an annual rate of 8% to triple in value if compounding

   (a) takes place quarterly? Round your answer to the nearest tenth of a year.

   (b) takes place continuously? Round your answer to the nearest tenth of a year. As usual, now work shown, no credit!