April 12, 2010   Name $\overline{\hspace{6cm}}$

The total number of points available is 145. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = x^3 - 8x^2 + 3, \quad -2 \le x \le 10$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

2. (20 points) Find a rational function $r(x)$ that has exactly three zeros, $x = -3$ $x = -1$ and $x = 1$, exactly one vertical asymptote at a number between -4 and 4, and has a horizontal asymptote $y = 3$.

   (a) Sketch the graph of your $r(x)$.

   (b) Find a symbolic representation of r.

3. (30 points) Consider the function $f(x) = (3x - 1)^2(x - 4)^2$.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Find all the critical points of f.

   (c) Apply the Test Interval Technique to find the sign chart for f' and use the information in the sign chart to classify the critical points of f. In other words, tell whether each one is the location of (a) a relative maximum, (b) a relative minimum, or (c) neither a relative max or min.

   (d) List the intervals over which f is increasing.

   (e) Discuss the concavity to f and find all the inflection points on the graph of f.

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4. (15 points) Find an equation for the line tangent to (the graph of) $f(x) = xe^{2x}$ at the point $(2, 2e^4)$. You can leave your answer in terms of e. In other words, you need not find decimal approximations.

5. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the difference between the object's temperature and that of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that
$$F(t) = T + Ae^{-kt},$$
where T is the air temperature, $72°F$, A and k are positive constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 11 minutes, the temperature of the coffee is $132°F$. What is the value of k?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

(d) After an hour, the coffee reaches a temperature of about $73°$. It has lost temperature at a rate of $(212 - 73)/60 \approx 7/3$ degrees per minute. Find a time t when the coffee is losing exactly $7/3$ degrees per minute.

6. (10 points) The population of the world in 1990 was 5 billion and the relative growth rate was estimated at 1.5 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2020.

7. (20 points) Consider the function $f(x) = x \ln(x^2 + 2)$.

   (a) Find $f'(x)$.

   (b) Find $f''(x)$.

   (c) Find the sign chart for $f''(x)$.

   (d) Find the intervals over which f is concave upwards.

8. (15 points) A rancher wants to fence in a rectangular grazing area along a river and then divide it in half with a fence down the middle perpendicular to the river to separate the cows from the sheep. What is the largest area that the rancher fence in if he has 12 miles of fencing material?