November 23, 2010 **Name** ______________________________

The total number of points available is 153. Throughout this test, **show your work. Throughout this test, you are expected to use calculus to solve problems. Graphing calculator solutions will generally be worth substantially less credit.**

1. (12 points) Find an equation for the line tangent to the graph of $f(x) = xe^{-2x+4}$ at the point $(2, f(2))$.

2. (12 points) Find an equation for the line tangent to the graph of $f(x) = x^2 \ln(x)$ at the point (e, e^2).

3. (12 points) A radioactive substance has a half-life of 27 years. Find an expression for the amount of the substance at time t if 20 grams were present initially.

4. (15 points) For each function f listed below, find the slope of the line tangent to its graph at the point $(0, f(0))$.

   (a) $f(x) = e^{e^x}$.

   (b) $f(x) = (x-1)^2 \cdot \ln(2x+1)$.

   (c) $f(x) = (1 + \ln(2x+1))^3$.

5. (10 points) For each function listed below, find a critical point.

   (a) $g(x) = x\ln(x)$.

   (b) $h(x) = (2x-3)e^{4x}$.

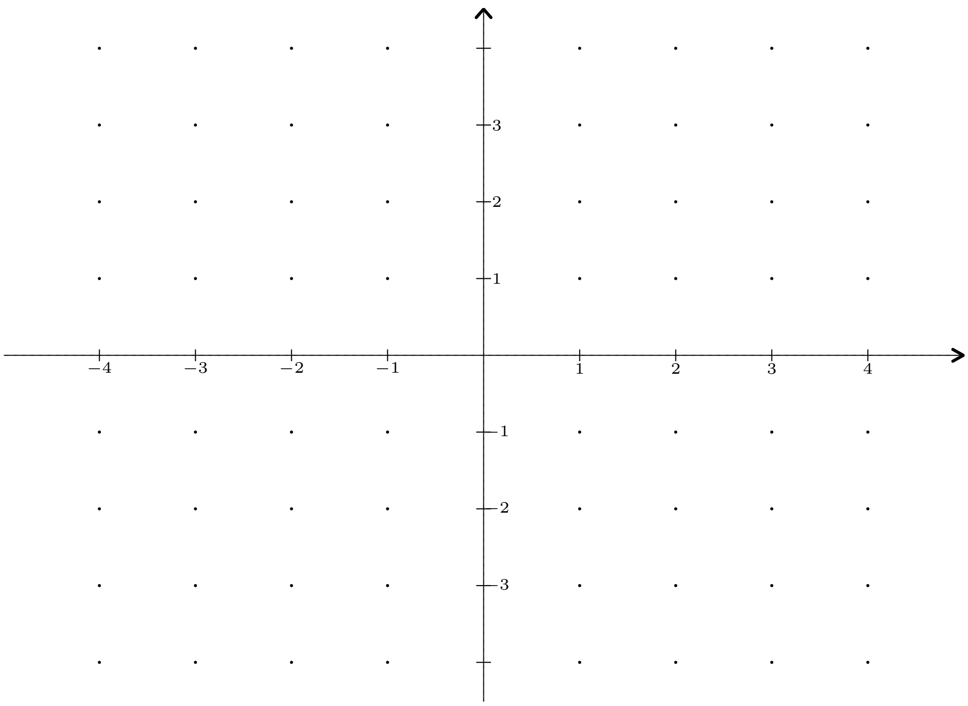
6. (15 points) Consider the function $f(x) = 1+9x+3x^2-x^3, \quad -2 \le x \le 6$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

7. (15 points) Consider the function $f(x) = \frac{(2x+3)(x-3)}{x(x-1)}$.

   (a) Build the sign chart for f

   (b) Find the vertical and horizontal asymptotes.

   (c) Use the information from the first two parts to sketch the graph of f.



8. (25 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $62°F$, A and k are positive constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 14 minutes, the temperature of the coffee is $112°F$. What is the value of k, correct to four places?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

(d) Take the derivative of $F(t)$ and show that the rate of change of temperature of the coffee is proportional to the difference between the room temperature and the coffee's temperature. That is, show that $F'(t) = l(F(t) - T)$ where l is a constant of proportionality. This is actually much easier than it might seem.

9. (25 points) Consider the function $f(x) = \ln(3x^2 + 1)$.

   (a) Find $f'(x)$.

   (b) Find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

   (c) Find $f''(x)$.

   (d) Find the sign chart for $f''(x)$.

   (e) Find the intervals over which f is concave upwards.

10. (12 points) Compound Interest.

   (a) Consider the equation $2000(1 + 0.03)^{4t} = 6000$. Find the value of t and interpret your answer in the language of compound interest.

   (b) Consider the equation $P(1 + 0.04)^{4 \cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

   (c) Consider the equation $Pe^{10r} = 2P$. Solve for r and interpret your answer in the language of compound interest.