April 13, 2011 **Name** ____________________

The problems count as marked. The total number of points available is 159. Throughout this test, **show your work.** Use calculus to work the problems. Calculator solutions which circumvent the ideas and techniques of the course will typically be worth about one fourth credit.

1. (12 points) Find an equation for the line tangent to the graph of $f(x) = xe^{-2x+4}$ at the point $(2, f(2))$.

2. (12 points) Find an equation for the line tangent to the graph of $g(x) = (x + \ln(x))^2$ at the point $(1, 1)$.

3. (25 points) Find a symbolic representation of a rational function $r(x)$ that has all the following properties:

   (a) i. It has exactly two zeros, $x = -1$ and $x = 2$.

   ii. It has two vertical asymptotes, $x = -2$ and $x = 3$.

   iii. It has $y = 3$ as a horizontal asymptote.

   (b) Find the derivative of your function.

   (c) Use the information in part b) to find the intervals over which your function is increasing.

4. (30 points) Consider the rational function r defined by

$$r(x) = \frac{(2x^2+3x-14)(x^2-1)}{(x^2+x-6)(x+1)^2}.$$

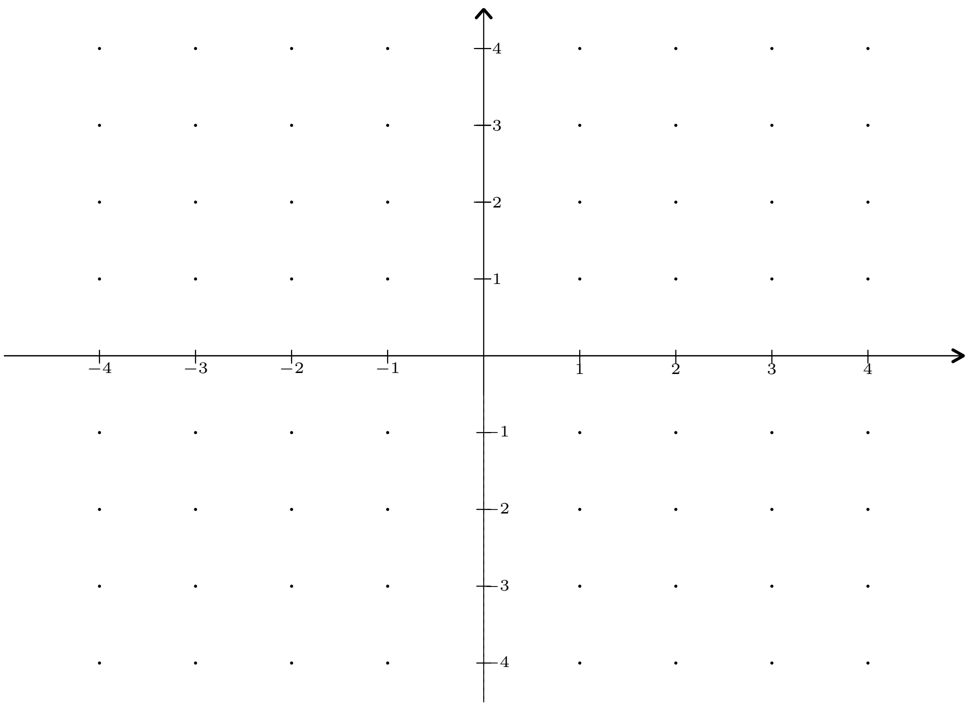
(a) Find the zeros of r.

(b) Find the vertical asymptotes of r.

(c) Does the function have any horizontal asymptotes? If so, what are they?

(d) Build the sign chart for r.

(e) Using precisely the information you found in the first three parts of the problem, sketch the graph of r.



5. (20 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 165e^{-0.02t}$$

where $m(t)$ is measured in grams.

(a) Find the mass at time $t = 0$.

(b) How much of the mass remains after 15 years?

(c) What is the half-life of the material?

(d) Find the rate of loss at $t = 1$ year.

6. (15 points) Four identical $x \times x$ square corners are cut from a 12×18 inch rectangular piece of metal, and the sides are folded upward to build a box.

(a) What is the volume of the box that results when the corners cut are 1×1.

(b) Let $V(x)$ denote the volume of the box when the $x \times x$ corners are removed. Find $V(2)$ and $V(3)$.

(c) What is the implied domain of V?

(d) Find $V'(x)$.

(e) Find the critical points of $V(x)$.

(f) What value of x makes the value of V maximum? Estimate within 0.01 the maximum value of V.

7. (20 points) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $f(x) = 16 - x^2$. For example two of the vertices of the rectangle could be $(-3, 0)$ and $(3, 0)$, both on the x-axis. Then the other two vertices would be $(3, f(3)) = (3, 16 - (3)^2) = (3, 7)$ and $(-3, f(-3)) = (-3, 7)$. In this case the area of the rectangle is $A = 6 \cdot 7 = 42$.

   (a) Now suppose we use $x = 2$ to get a vertex. Then one vertex is $(2, 0)$. What are the other three vertices?

   (b) What is the area of the rectangle determined by this choice $x = 2$?

   (c) How does the area depend on x. In other words, if R is the rectangle determined by x, (and $-x, f(x), f(-x)$), what is the area $A(x)$ of R?

   (d) What choices of x give rise to rectangles? In other words, what is the domain of the function in part 3.

   (e) What are the dimensions of such a rectangle with the greatest possible area?

8. (10 points) Consider the line $y = 4$ and the point $P = (3, 2)$. For each real number x, let $D(x)$ denote the distance from the point P to the point $(x, 4)$ on the line. Find $D(x)$. State in words what it means when $D'(x) = 0$. In other words, what is the geometric meaning, not simply that x is a critical point of D. Find a critical point of D. Why does this value make sense? Write a complete sentence about your reasoning.

9. **Compound Interest.** (15 points)

   (a) Find the time required for a 12% investment compounded quarterly to quadruple in value. Is there a way to apply the 'rule of 72' to estimate this answer.

   (b) Given that a certain investment compounded continuously has taken exactly 14 years to triple its value, what was the rate of interest?

   (c) Now change (b) so the compounding is annual and work the same problem again. Explain why this rate is higher than in part (b).