November 17, 2011 **Name** ______________________________

The total number of points available is 154. Throughout this test, **show your work.**

1. (10 points) Find an equation for the line tangent to the graph of $y = \ln(x^2+1)$ at the point $(1, \ln(2))$.

2. (10 points) Find an equation for the line tangent in slope-intercept form to the graph of $y = e^{4x-2}$ at the point $(1/2, 1)$.

3. (15 points) Consider the function $f(x) = (x^2 - 4x + 4)e^{2x}$.

   (a) Use the product rule to find $f'(x)$.

   (b) List the critical points of f.

   (c) Construct the sign chart for $f'(x)$.

   (d) Write in interval notation the interval(s) over which f is increasing.

4. (20 points) The product of two positive numbers is 1200. What is the smallest possible value of 3 times the first plus the second?

5. (12 points) A skull from an archeological dig has one-twelfth the amount of Carbon-14 it had when the specimen was alive.

   (a) Recall that the half-life of Carbon-14 is 5770 years. Find the decay constant k.

   (b) What is the age of the specimen? Round off your answer to the nearest multiple of one hundred years.

6. (12 points) Compound Interest.

   (a) Consider the equation $3000(1+0.03)^{4t} = 6000$. Find the value of t and interpret your answer in the language of compound interest.

   (b) Consider the equation $P(1+0.04)^{4\cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

   (c) Consider the equation $Pe^{10r} = 3P$. Solve for r and interpret your answer in the language of compound interest.

7. (25 points) Consider the function $f(x) = \ln(x^2 + 4)$.

   (a) Find the domain of f.

   (b) Find both $f'(x)$ and $f''(x)$.

   (c) Build the sign chart for $f''(x)$.

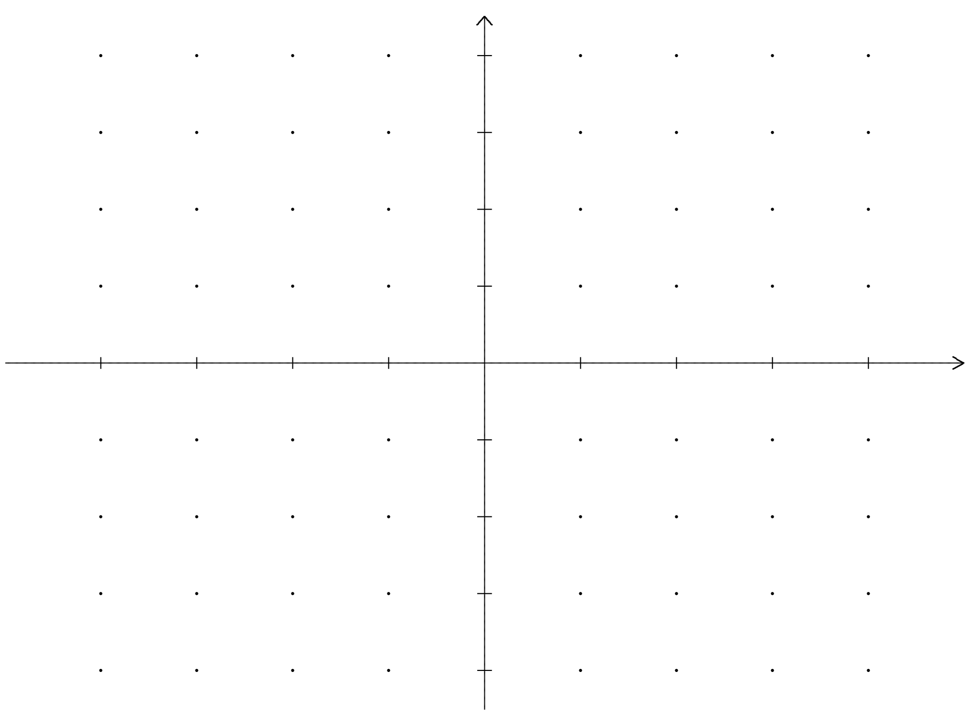
   (d) Does f have an absolute maximum or and absolute minimum? Is so, which? ... and where?

   (e) Find an interval over which f is increasing. As usual, to get credit for this, you must show your work.

   (f) Does f have any inflection points?

   (g) Use the information from part c. to find the intervals where f is concave upwards.

   (h) Based on the information you found above, sketch the graph of f. For any credit, your graph MUST be consistent with the sign charts you found.



8. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the difference between the object's temperature and that of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $68°F$, A and k are positive constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 8 minutes, the temperature of the coffee is $136.6°F$. What is the value of k?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

9. (30 points) Spreading of a rumor. Three hundred college students attend a lecture of the Dean at which she hints that the college will become coed. The rumor spreads according to the logistic curve

$$Q(t) = \frac{3000}{1 + Be^{-kt}},$$

where t is measured in hours.

(a) Compute the parameter B.

(b) How many students attend the college? Hint: the question is not 'how many attended the lecture?'

(c) Two hours after the speech, 600 students had heard the rumor. How many students had heard the rumor after four hours?

(d) How fast is the rumor spreading after four hours?

(e) After how many hours will 2000 students have heard the rumor?