

April 6, 2001

Name _____

In the first five problems, each part counts 7 points (total 35 points) and the final two problems count as marked. This test will be scored as if it were an 80 point test. There are 100 points available. Good luck.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Let $f(x) = e^{2x^2+x}$. What is $f'(0)$?

- (A) 0 (B) 1 (C) e^2 (D) $2e^2$ (E) $2e^3$

Solution: $f'(x) = (4x + 1)e^{2x^2+x}$ so $f'(0) = 1 \cdot e^0 = 1$.

2. Let $f(x) = \ln(x^2)$. What is $f'(e)$?

- (A) 0 (B) 2 (C) 4 (D) $2e^{-1}$ (E) $2e^2$

Solution: $f'(x) = 2x/x^2$ so $f'(e) = 2e/e^2 = 2e^{-1}$.

3. Which of the following is closest to a solution to $2e^{x+1} = 2001$?

- (A) 5.74 (B) 5.91 (C) 6.60 (D) 6.91 (E) 7.21

Solution: Take natural log of both sides to get $x + 1 = \ln(2001/2) = \ln(1000.5) = 6.908$. Therefore, $x = 5.908 \approx 5.91$.

4. Population Growth. The population of a town increases according to the model

$$P(t) = 2200e^{0.04t}$$

where t is measured in years with $t = 0$ corresponding to 1990. Use the model to approximate the population in 2001.

- (A) 3401 (B) 3411 (C) 3416 (D) 3466 (E) 3487

Solution: $P(11) = 2200e^{0.04 \cdot 11} \approx 2200 \cdot 1.5527 \approx 3416$.

5. A total of \$10,000 is invested at an annual rate of 9%. Find the balance after 5 years if it is compounded quarterly.

- (A) \$15,605 (B) \$15,683 (C) \$15,720 (D) \$15,818 (E) \$15,988

Solution: $A = 10000(1 + .09/4)^{20} \approx 15605$.

On all the following questions, **show your work**.

6. (15 points) Suppose that \$300 is deposited into an account with an annual percentage rate of 8%. How long does it take this investment to double, assuming the compounding takes place

(a) quarterly? Round your answer to the nearest tenth of a year.

Solution: Solve the equation $600 = 300(1 + 0.08/4)^{4t}$ for t to get $t = 8.8$ years.

(b) continuously? Again, round your answer to the nearest tenth of a year.

Solution: Again solve the equation $600 = 300e^{0.08t}$ for t to get $t \approx 8.664 \approx 8.7$ years.

7. (20 points) A radioactive substance has a half-life of 33 years.
- (a) Use the exponential decay model to write an equation which shows that after 33 years, a sample with 200 grams of radioactivity has only 100 grams left.
- (b) Use the fact that there are **initially** 200 grams of radioactivity to solve for one of the functions parameters.
- (c) Use the information in (a) and (b) to solve for the constant k in the function.
- (d) How many years must elapse before the amount of radioactivity is down to 25 grams.

8. (15 points) Find the derivatives of each of the following functions.

(a) $f(x) = \ln(x^2 - x + 1)$

(b) $g(x) = e^{x^2-3x+5}$

(c) $h(x) = 2xe^x$

9. (15 points) Discuss the intervals of concavity of the function $f(x) = x^2 + \ln(x^2)$.