Math 1120 **Calculus** **Section 3** **Test 4**

April 24, 2002 Your name ____________________

All the problems are marked with their value. The total number of points available is 114. Throughout the test, **show your work.**

1. (10 points) Find the interval(s) over which the function $f(x) = 2x^3 + 3x^2 - 36x + 17$ decreasing?

   Solution: Differentiate and factor to get $f'(x) = 6x^2+6x-36 = 6(x-2)(x+3)$, so the critical points are $x = 2$ and $x = -3$. Use the Test Interval Technique to solve the inequality $f'(x) < 0$. Only the interval $[-3, 2]$ works.

2. (10 points) Find the absolute maximum value of the function $f(x) = e^{-x^2+x}$ on the interval $-2 \le x \le 3$.

   Solution: Find $f'(x)$ first and then the critical points that are between -2 and 3. $f'(x) = (-2x + 1)e^{-x^2+x}$, so there is one critical point $x = 1/2$ and two endpoints to check: $f(-2) = e^{-6} \approx 0.00247; f(3) == e^{-6} \approx 0.00247$; and $f(1/2) = e^{1/4} \approx 1.284$, so the absolute maximum is $f(1/2)$, which of course occurs at $x = 1/2$.

3. (10 points) Let $g(x) = \ln((2x - 3)(2x + 1)(x + 3)(x - 5))$. Find the (implied) domain of g.

   Solution: The domain of $\ln z$ is the set $z > 0$ so we need to solve the inequality $(2x-3)(2x+1)(x+3)(x-5) > 0$. We do this by the Test Interval Technique. We get $(-\infty, -3) \cup (-1/2, 3/2) \cup (5, \infty)$.

4. (10 points) Sketch an example of a continuous function $f(x)$ that has domain $[-4, 4]$, and satisfies the following requirements.

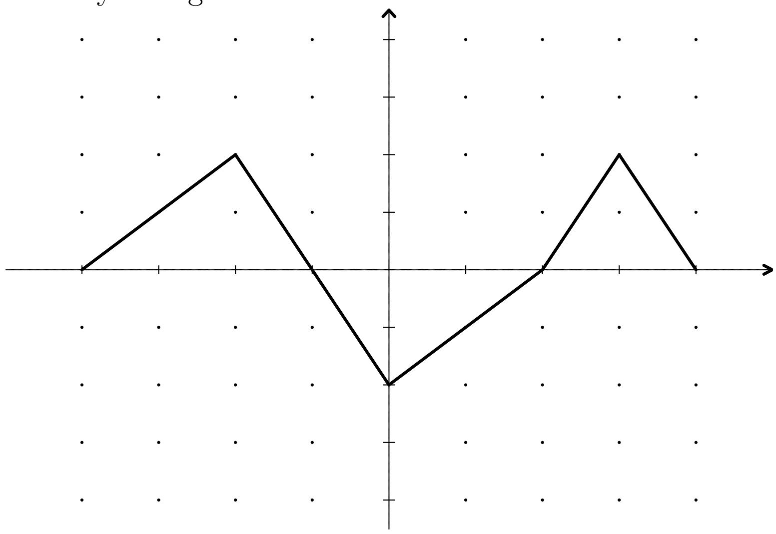
   (a) $f(-4) = f(-1) = f(2) = 0$.

   (b) f is increasing on $[-4, -2]$.

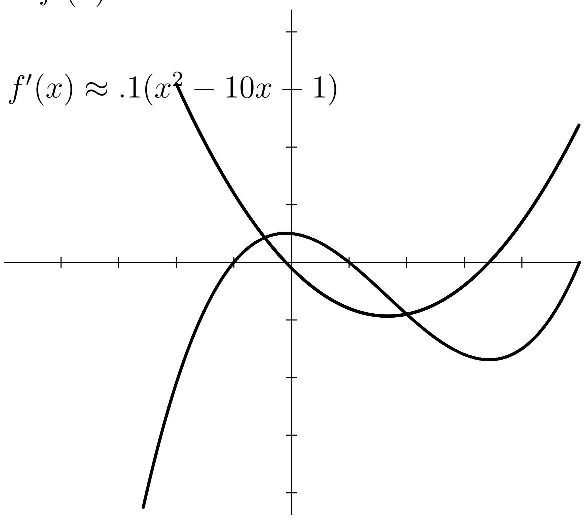
   (c) f has a singular point at $x = 3$.

   (d) f has a relative maximum at $x = 3$ and a value of 2 at $x = 3$.
   Solution: Such a function is plotted below.

   Use the coordinate system given.



5. (10 points) Let f be the function whose graph is shown below. On the same axes, plot the graph of $f'(x)$.



6. (10 points) Sketch an example of a function $f(x)$ that has domain $[-4, 4]$, and satisfies the following requirements. Please note: this problem has been slightly modified from the original, which interchanged the 1 and the 2 in the first two conditions.

   (a) $\lim_{x \to -2^+} f(x) = 1$.

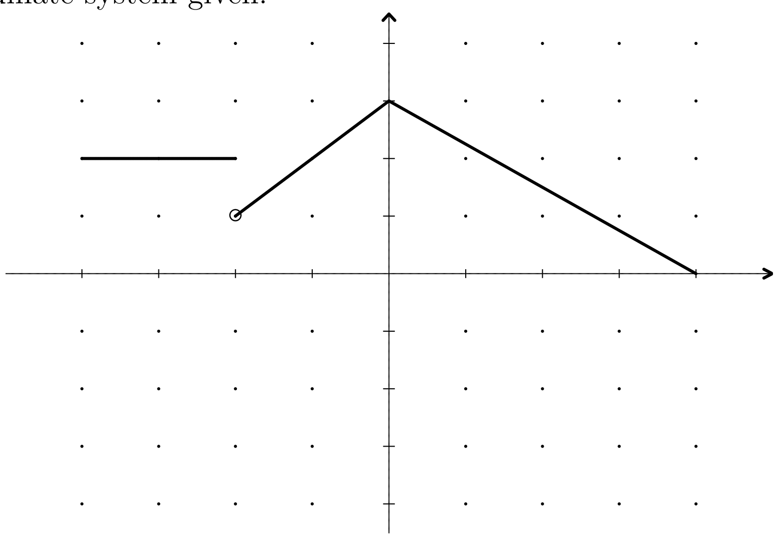
   (b) $\lim_{x \to -2^-} f(x) = 2$.

   (c) $f(2) = 0, f(0) = 3$

   (d) f is linear on the interval $[0, 4]$.

   (e) f has an absolute maximum at $x = 0$.

   Use the coordinate system given.



7. (10 points) Solve the equation $2 + 3 \cdot 5^{2x+1} = 77$.

   Solution: Subtract 2 from both sides to get $3 \cdot 5^{2x+1} = 75$. Then divide by 3 to get $5^{2x+1} = 25 = 5^2$. Since the bases are the same, the exponents must also be the same. Hence $2x + 1 = 2$ and $x = 1/2$.

8. (10 points) Compound Interest. Find the time required for an 8% investment compounded quarterly to triple.

   Solution: We need to solve the equation

$$3 = 1\left(1 + \frac{0.08}{4}\right)^{4t}.$$

   Take logs of both sides to get $4t = \frac{\ln 3}{\ln 1.02}$. Divide both sides by four to get $t = \frac{\ln 3}{4 \ln 1.02} \approx 13.87$ years.

9. (12 points) Compute the following limits.

   (a) $\lim_{x\to\infty} \dfrac{3x^3 - 5x^2 + 10}{2x^3 + 10x - 5}$.
   Solution: Since both polynomials have degree 3, the limit is the ration of the leading coefficients, 3/2.

   (b) $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$.
   Solution: Factor the denominator and cancel the common terms to get $\lim_{x\to 2} \dfrac{1}{x+2} = 1/4$

   (c) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$.
   Solution: Rationalize the numerator (or factor the denominator) to get $\lim = 1/6$.

10. (12 points) Find the following derivatives.

    (a) $\frac{d}{dx} xe^x$
    Solution: Use the product rule to get $\frac{d}{dx} xe^x = e^x + xe^x$

    (b) $\frac{d}{dx} \frac{\ln(x)}{x}$
    Solution: Use the quotient rule to get $\frac{d}{dx} \frac{\ln(x)}{x} = \frac{1-\ln x}{x^2}$

    (c) $\frac{d}{dx} e^{\ln(x^5+x^2-2x)}$
    Solution: The function is just the exponent since the two functions being composed are inverses of each other. Therefore, $\frac{d}{dx} e^{\ln(x^5+x^2-2x)} = 5x^4 + 2x - 2$.

11. (10 points) Let

$$f(x) = \begin{cases} -x/2 + 2 & \text{if } x \leq -1 \\ x + 3 & \text{if } -1 < x < 3 \\ x^2 - 5x & \text{if } 3 \leq x \end{cases}$$

Find an equation for the line tangent to the graph of f at the point $(4, -4)$.

Solution: Note that near $x = 4$, $f'(x) = 2x - 5$, so $f'(4) = 8 - 5 = 3$. Thus the line is given by $y - 4 = 3(x + 4)$, which is equivalent to $y = 3x + 16$.