

1. In the city of OZ, anyone 40 years or older always tells the truth and anyone under 40 never tells the truth. A census taker knocks on the married couple's door. The husband opens the door. 'I am the census taker,' says the visitor, 'and I need information about you and your wife. Which, if either of you, are under the age of 40?' 'We are both under 40!' said the husband angrily as he slammed the door. How did the census taker sort out the facts.

Solution: Let h and w denote the ages of the husband and wife respectively. Then $h < 40$, otherwise the husband would have been telling the truth, which he cannot do unless his is at least 40. But if he is lying, he must not be lying about his own age. He must therefore be lying about his wife's age. We are forced to conclude that $h < 40$ and $w > 40$.

2. Lucy, Minnie, Nancy, and Opey ran a race. Each was asked how the race turned out. Their replies:

Lucy: Nancy won. Minnie was second.

Minnie: Nancy was second. Opey was third.

Nancy: Opey was last. Lucy was second.

If each girl made exactly one true statement, who won the race?

Solution: Let O_3 denote the statement: Opey finished third. In general, let T_i denote the statement that the person whose name starts with the letter T finished in position i . Then Lucy's statement can be denoted $N_1 \vee M_2$ since exactly one of Lucy's assertions is true. Likewise Minnie's statement is $N_2 \vee O_3$, and Nancy's is $O_4 \vee L_2$. Note that both Lucy's assertions contradicts N_2 . This can only happen if N_2 is false, which means O_3 must be true. Since exactly one of O_4 and L_2 is true, it must be that L_2 is true. This implies that N_1 is true and we have $N_1 L_2 O_3 M_4$.

3. The three statements below are all true.

1. Either A is not guilty or B is guilty.

2. Either B is not guilty or C is not guilty.

3. If A is guilty, then B and C are both guilty.

Where does the guilt lie?

Solution: Let a denote the statement A is guilty, b , the statement B is guilty, and c , the statement C is guilty. Then the three conditions can be written 1. $\bar{a} \vee b$, 2. $\bar{b} \vee \bar{c}$, and 3. $a \rightarrow (b \wedge c)$. But notice that $a \rightarrow (b \wedge c) \Leftrightarrow \bar{a} \vee (b \wedge c)$. Since $b \wedge c$ and $\bar{b} \vee \bar{c}$ contradict each other, we conclude that \bar{a} . But the two possibilities $\bar{a}\bar{b}c$ and $\bar{a}b\bar{c}$ are both viable.

4. A says “ B is a liar or C is a liar”;
 B says “ A is a liar”;
 C says “ A is a liar and B is a liar”. Who is telling the truth?

Solution: The key here is to note that if a person is not a liar, then his statement must be true and if he is a liar, his statement must be false. Let a denote A is a liar, b : B is a liar, and C : C is a liar. Then A 's statement is $b \vee c$, B 's is a and C 's is $a \wedge b$. Let's suppose that C tells the truth. Then $a \wedge b$, which means that both A and B are liars. So both A 's and B 's statements are false. To say a is false is to say A is not a liar, a contradiction. Therefore C is a liar and $a \wedge b$ is false. Thus $\bar{a} \vee \bar{b}$. This is consistent with A 's statements. Therefore, we have $\bar{a}bc$. In other words, A is telling the truth and B and C are both lying.

5. What logical conclusion can be drawn from:
 A says both B and C tell the truth;
 B says A tells the truth;
 C says A and B are both liars?

Solution: If either A or B tells the truth, then C must as well, a contradiction. Thus A and B both lie and C tells the truth.

6. Three men named Arnold, Brown, and Clark hold the positions of shipper, driver, and manager in a certain company.
If Clark is the shipper, Brown is the driver.
If Clark is the driver, Brown is the manager.
If Brown is not the shipper, Arnold is the driver.
If Arnold is the manager, Clark is the driver.
Who held each of the positions?

Solution: If Brown is not the shipper, then Arnold is the driver, in which case Brown is the manager and Clark is the shipper. But if Clark is the shipper, then Brown is the driver, a contradiction.

So Brown is the shipper. Now if Clark is the driver, then Brown is the manager, a contradiction. Thus Clark is the manager and Arnold is the driver. This problem can be done by brute force using the algebra of propositions as well.

7. The King is about to die. To determine who will succeed him as king, He sends messengers throughout the land seeking the three smartest people. Finally they are found. He gives them a task to see which one is the wisest. He tells them, "I will seat you in a triangle so that each of you faces the other two. After you are blindfolded I will paint a dot on each of your foreheads. Each

dot will be red or green so that there can be any combination of red and green dots, for example, 1 red and 2 greens, or all red, etc. When I remove the blindfolds each of you must raise your hand if you see any green dots, i.e. 1 or 2 dots. As soon as you have figured out what color your own dot is, lower your hand and tell me." So he seats them, blindfolds them, and then paints a green dot on all three foreheads. When the blindfolds are removed, all three hands go up. After a long pause, one hand comes down and the man says, "Your highness, I have a green dot." How did he know?

Solution: Call him Abe. Abe has a green. Suppose not. Then Abe's dot is red and both the other two would have known immediately that he had a green dot, otherwise, we would not have seen all three hands go up. Since neither of the other two claimed to have a green dot, it can only be because Abe's dot is green.