Throughout we use both the notations $\binom{n}{r}$ and C_r^n for the number $\frac{n!}{(n-r)!r!}$.

1. Ten points are distributed around a circle. How many triangles have all three of their vertices in this 10-element set?

2. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set. Let D denote the set of all four-digit numbers that can be built using the elements of S as digits and allowing repetition of digits.

   (a) Find the number of four element subsets of S

   (b) What is $|D|$? In other words, how many four-digit numbers are there?

   (c) How many elements of D have four different digits?

   (d) How many elements of D have exactly three different digits?

   (e) How many even numbers belong to D?

3. The following problems are related.

   (a) What is the value of $\frac{7!}{(7-3)!3!}$?

   (b) How many 3-element subsets does the set $\{A, B, C, D, E, F, G\}$ have?

   (c) How many solutions are there to

   $$x + y + u + v = 4$$

   where x, y, u, and v are nonnegative integers. For example, $(2, 1, 0, 1)$ is such a solution.

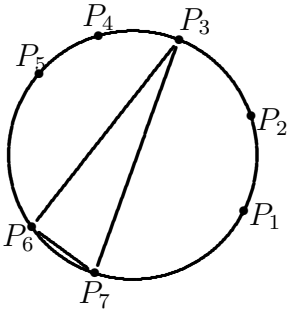
   (d) How many solutions does

   $$x + y + u + v = 8$$

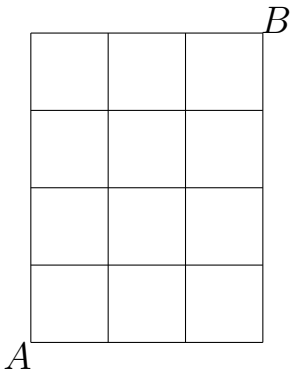
   have subject to the condition that each of the variables is a positive integer?

   (e) How many ways can a 3-person committee be selected from a 7-member club?

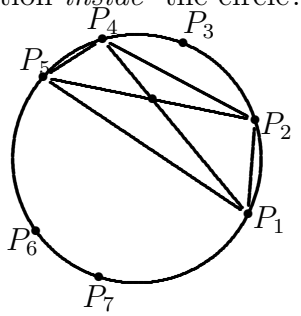
   (f) Let $P_1, P_2, P_3, P_4, P_5, P_6, P_7$ be seven points distributed around a circle. How many triangles have all three vertices in the set.



(g) How many paths of length 7 are there from A to B in the grid below?



(h) Seven points are distributed around a circle. All pairs of them are joined by a secant line. What is the largest possible number of points of intersection *inside* the circle?



(i) What is the coefficient of x^3 in the expanded form of $(x+1)^7$?

(j) What is the third entry of the seventh row of Pascal's triangle?

(k) How many numbers can be expressed as a sum of four distinct members of the set $\{1, 2, 4, 8, 16, 32, 64\}$?

November 14, 2005 2:37 P.M.