Throughout we use both the notations $\binom{n}{r}$ and C^n_r for the number $\frac{n!}{(n-r)!r!}$.

1. A *falling* number is an integer whose decimal representation has the property that each digit except the units digit is larger than the one to its right. For example 96521 is a falling number but 89642 is not. How many n-digit falling numbers are there, for $n = 1, 2, 3, 4, 5, 6, 7, 8,$ and 9? What is the total number of falling numbers of all sizes?

2. Cyprian writes down the middle number in each of the $\binom{9}{5} = 126$ five-element subsets of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Then he adds all these numbers together. What sum does he get?

3. Counting sums of subset members.

   (a) How many number can be expressed as a sum of two or more distinct elements of the set $\{1, -3, 9, -27, 81, -243\}$?

   (b) How many numbers can be expressed as a sum of two or more distinct members of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

   (c) How many numbers can be expressed as a sum of four distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

   (d) How many numbers can be expressed as a sum of two or more distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

4. How many of the first 242 positive integers are expressible as a sum of three or fewer members of the set $\{3^0, 3^1, 3^2, 3^3, 3^4\}$ if we are allowed to use the same power more than once. For example, $5 = 3 + 1 + 1$ can be represented, but 8 cannot. Hint: think about the ternary representations.

5. How many integers can be expressed as a sum of two or more different members of the set $\{0, 1, 2, 4, 8, 16, 32\}$?

6. John has 2 pennies, 3 nickels, 2 dimes, 3 quarters, and 8 dollars. For how many different amounts can John make an exact purchase (with no change required)?

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