Math 1165 **September 22, 2005** **Test 1**

Your name ________________________________

There are 134 points available on this test. You must **show all your work**.

1. (20 points) Let P denote the compound proposition defined by $P : (p \to q) \to r$ and let $Q : p \to (q \to r)$. Test the associativity of $\to$ by determining whether P and Q are logically equivalent.

2. (24 points) Determine the truth value of the following statements if the universe of discourse of each variable is the set of real numbers.

___1. $\forall x \exists y (x = y^2)$

___2. $\forall x \exists y ((x + y = 2) \wedge (2x - y = 1))$

___3. $\exists x (x^2 = -1)$

___4. $\forall x \neq 0 \exists y (xy = 1)$

___5. $\exists x \exists y (x + y \neq y + x)$

___6. $\forall x \exists y (x + y = 1)$

___7. $\exists x (x^2 = 2)$

___8. $\exists x \forall y \neq 0 (xy = 1)$

___9. $\forall x \forall y \exists z (z = \frac{x+y}{2})$

___10. $\exists x \exists y ((x + 2y = 2) \wedge (2x + 4y = 5))$

___11. $\exists x \forall y (xy = 0)$

___12. $\forall x \exists y (x^2 = y)$

3. (20 points) Let D denote the set of all real numbers and let P and Q denote the two-place predicates on D defined by $P(x, y) : x \leq y$ and $Q(x, y) : y \leq x$. Find the truth value of each of the compound propositions.

   (a) $\forall x \forall y (P(x, y) \wedge Q(x, y)) \rightarrow x = y$.

   (b) $\forall x \forall y (P(x, y) \rightarrow -Q(x, y))$.

   (c) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$.

   (d) $\forall x \forall y (P(x, y) \vee Q(x, y)) \rightarrow x \neq y$.

   (e) $\forall x \forall y \; x \neq y \leftrightarrow (-P(x, y) \vee -Q(x, y))$.

4. (20 points) Notice that

$$\frac{1}{1\cdot 2} = \frac{1}{2} \tag{1}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3} = \frac{2}{3} \tag{2}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4} = \frac{3}{4} \tag{3}$$

(a) List the next two equations suggested by the pattern.

(b) Given that the three equations above are the 1^{st}, 2^{nd}, and 3^{rd}, write the n^{th} equation of the sequence.

(c) Use mathematical induction to prove that the n^{th} equation is true for all positive integer values of n.

5. (20 points) Prove that $4^n - 1$ is divisible by 3 for all $n \geq 1$.

6. (10 points) In a group of 100 students, the following facts are known:

   - 50 take accounting,
   - 40 take biology,
   - 35 take chemistry,
   - 12 take both accounting and biology,
   - 10 take accounting and chemistry,
   - 11 take chemistry and biology, and
   - 5 take all three subjects.

   How many take none of the three subjects?

7. (20 points) Let Z denote the set of all integers. Classify each of the following functions from Z to Z as one-to-one and onto, one-to-one and not onto, onto and not one-to-one, neither onto nor one-to-one. Prove your answers.

   (a) Let $f(n) = \begin{cases} 2n & \text{if } n \geq 0 \\ -2n-1 & \text{if } n < 0 \end{cases}$

   (b) Let $f(n) = \begin{cases} n-1 & \text{if } n \geq 1 \\ n+1 & \text{if } n \leq 0 \end{cases}$

   (c) $f(n) = -n$

   (d) $f(n) = |n|$