# MATH 1241 FALL 2003
# COMMON FINAL EXAMINATION
# PART I

Name:______________________ Instructor:______________________

Student ID #:____________________ Section/Time:__________________

This exam is divided into three parts. You have three hours for the entire test. You have only one hour to finish Part I **without using any calculator** and submit the provided answer sheet at 9:00 am. You may start working on the other two parts of the test whenever you're done with Part I but still no calculators are allowed. After collecting the answer sheets to Part I, your instructor will announce that the calculators are allowed to work on Part II and Part III of the exam.

These pages contain Part I which consists of 10 multiple choice questions. It must be done without using any calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

**At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.**

## PART I (NO CALCULATOR ALLOWED)

1. If $f(x) = \sqrt{5x^2+49}$ then $f'(x)$ is

A. 7

B. $\frac{1}{2\sqrt{5x^2+49}}$

C. $\sqrt{10x}$

D. $\frac{2}{3}(5x^2+49)^{3/2}$

E. $\frac{5x}{\sqrt{5x^2+49}}$

2. If $f(x) = \sin^3(5x+2)$, then $f'(x)$ is

A. $3\sin^2(5x+2)$

B. $15\cos^2(5x+2)$

C. $15\sin^2(5x+2)\cos(5x+2)$

D. $3\sin^2(5x+2)\cos(5)$

E. $3\cos^2(5x)$

3. If $f(x) = 9x^{2/3} - 24x^{1/3} + 5$, then $f''(8) =$

A. $\frac{1}{24}$

B. $\frac{5}{12}$

C. 1

D. 6

E. $\frac{50}{3}$

4. An equation for the tangent line to the curve $y = e^{x^2}$ at the point $(1, e)$ is

A. $2ex - y = e$

B. $ex - y = e$

C. $(\ln 2)x - y = e$

D. $ex + y = e^2$

E. $2x - ey = 1$

5. The derivative of $\ln(3x + 7)$ at $x = -2$

A. does not exist

B. is $0$

C. is 1

D. is 3

E. is $e^3$

6. The linearization $L(x)$ of the function $\arctan x$ at 0 is

A. $L(x) = \frac{x}{1+x^2}$

B. $L(x) = \frac{\pi}{4} + x$

C. $L(x) = 1 + \frac{1}{2}x$

D. $L(x) = x$

E. $L(x) = x - \frac{1}{3}x^3$

7. $\lim_{x \to 2} \frac{x^2+x-6}{4x-8}$

A. equals $\frac{3}{4}$

B. equals 0

C. equals $\frac{3}{2}$

D. equals $\frac{5}{4}$

E. does not exist

8. If $f'(x) = \sin x + x^3$ and $f(0) = 4$ then $f(x)$ equals

A. $-\cos x + 3x^2$

B. $-\tan x + 3x^2 + 5$

C. $\cos x + \frac{1}{4}x^4 + 4$

D. $-\cos x + \frac{1}{4}x^4 + 5$

E. $\csc x + x^{-3} + \frac{5}{2}$

9. In a search for a root of a function $f$, the starting value for Newton's Method is $x_1 = 3$. If $f(3) = 2.4$ and $f'(3) = -0.8$, then one iteration of the algorithm yields $x_2 =$

A. $-2.4$

B. $-0.4$

C. 3.33

D. 4.92

E. 6

10. The function $f(x) = x \ln x$ is

A. decreasing on $(0, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$

B. increasing on $(0, \infty)$

C. decreasing on $(0, \infty)$

D. increasing on $(0,1)$ and decreasing on $(1, \infty)$

E. decreasing on $(0, \frac{1}{e})$ and $(e, \infty)$ and increasing on $(\frac{1}{e}, e)$

# MATH 1241
# COMMON FINAL EXAMINATION
# PART II

FALL 2003

Name:__________________________ Instructor:______________________

Student ID #:______________________ Section/Time:__________________

These pages contain Part II which consists of 15 multiple choice questions. You are allowed to use a calculator to this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

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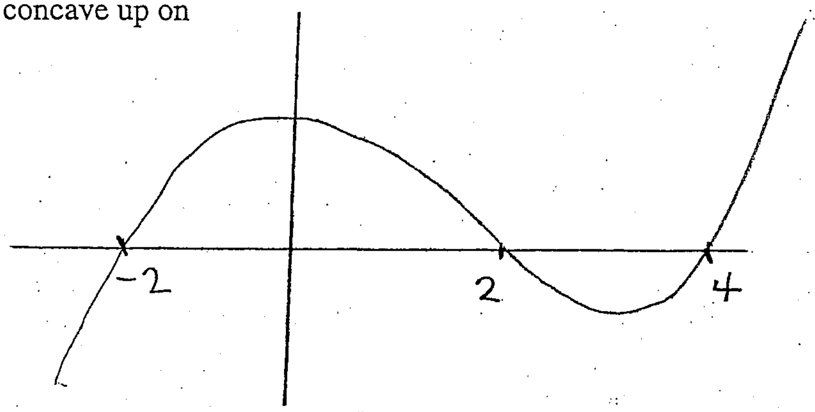
## PART II (CALCULATORS PERMITTED)

1. The graph on the right is a graph of $f''$, the second derivative of the function $f$. Based on this graph, the original function $f$ is concave up on




A. no interval

B. $(-2, 2)$

C. $(1, \infty)$

D. $(-2, 2)$ and $(4, \infty)$

E. $(-\infty, 0)$ and $(3, \infty)$

2. If the graph of a function $f$ consists of the bottom half of the circle $x^2 + y^2 = 1$, then $f(0.6)$ is

A. $-3.2$

B. $-2$

C. $-1.6$

D. $-0.8$

E. $0.8$

3. If $f(x) = \sqrt{x+2}$ and $g(x) = \frac{4}{x-3}$, the domain of $g \circ f$ is

A. $(0, \infty)$

B. $\{x : x \geq -2 \text{ and } x \neq 7\}$

C. $\{x : x > 0 \text{ and } x \neq 3\}$

D. all $x \neq -3$

E. $[-2, 9]$

4. An example of a function $f$ such that $f(1) = 2$ , $f(3) = 0$, and $f'(x) > 0$ for all $x$

A. is $f(x) = -x + 3$

B. is $-\frac{1}{4}x^2 + \frac{9}{4}$

C. is $f(x) = -1 + \sin \frac{\pi}{2}x$

D. cannot exist

E. is $f(x) = \frac{3}{x} - 1$

5. Let $f$ be a function whose <u>derivative</u> is $f'(x) = (x^6 + 1)^{-1}$. [Do not try to find a formula for $f$.] Which of the following statements is correct?

A. $f$ is increasing on $(-\infty,\infty)$ and concave upward on $(0,\infty)$.

B. $f$ is increasing on $(-\infty,\infty)$ and concave upward on $(-\infty, 0)$.

C. $f$ is decreasing on $(-\infty,0)$ and concave upward on $(0,\infty)$.

D. $f$ is decreasing on $(-\infty,\infty)$.

E. $f$ is concave upward on $(-\infty,\infty)$.

6. Let $h(x) = \cos(g(x))$. If $g(0) = \frac{\pi}{6}$ and $g'(0) = 3$, then $h'(0)$ equals

A. $-3$

B. $-1.5$

C. 0

D. $\frac{3\sqrt{3}}{2}$

E. $\frac{\pi}{2}$

7. If $h(x) = \frac{g(x)}{x^2}$ then $h'(x)$ equals

A. $\frac{g'(x)}{2x}$

B. $\frac{g(x+h)}{(x+h)^2} - \frac{g(x)}{x^2}$

C. $\frac{x^2}{g(x)}$

D. $\frac{xg'(x)-2g(x)}{x^3}$

E. $\frac{2xg(x)-x^2g'(x)}{g(x)^2}$

8. If $2x^2y - 3y^2 + 8 = 0$ is the equation of a certain curve, then the slope of the tangent line to this curve at the point (1,2) is
A. $-8$

B. $-6$

C. 0

D. 0.8

E. 1.6

9. For $x$ in the interval $[0, 1]$, the point on the graph of the function $f(x) = 1 + 4x - 3\sqrt[3]{x}$ with the smallest $y-$ coordinate is

A. (0,0)

B. $(\frac{1}{8}, 0)$

C. (0,1)

D. $(\frac{1}{9}, -\frac{2}{9})$

E. $(1, -1)$

10. A particle moves along the curve $y^2 = x^3$. As it reaches the point $(2, 2\sqrt{2})$, the $y$-coordinate is increasing at the rate of 4 cm/sec. At that instant, the $x$-coordinate is increasing at the rate of

A. $6\sqrt{2}$ cm/sec

B. 4 cm/sec

C. $\frac{4}{3}\sqrt{2}$ cm/sec

D. 3 cm/sec

E. $\sqrt{2}$ cm/sec

**Use the following table of values for the functions $f$, $g$, $f'$, $g'$ to answer questions 11 and 12:**

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 6 |
| 2 | 1 | 8 | 5 | 7 |
| 3 | 7 | 2 | 7 | 9 |

11. If $h(x) = f(g(x))$, then the value of $h'(1)$ is

A. 8

B. 24

C. 30

D. 35

E. 40

12. If $t(x) = 5x^2 g(x)$, then the value of $t'(3)$ is

A. 90

B. $90x$

C. 270

D. 405

E. 465

13. The positive number $x$ such that the sum $2x + \frac{1}{2x}$ is as small as possible is

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

E. 2

14. Let $f$ be a function such that

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = -1.$$

Considering the following statements:

I. $f$ is continuous at $x = 2$

II. $f'(2) = -1$

III. $f$ has a local minimum at $x = 2$

A. only I must be true
B. only I and II must be true
C. only II must be true
D. only II and III must be true
E. only III must be true

15. The distance $s$ in meters traveled by an object $t$ seconds after it begins moving is given by the equation

$$s = 7t^3 - 12t.$$

The average velocity of the object during the time interval from $t = 5$ sec to $t = 10$ sec is

A. 513 m/sec

B. 834 m/sec

C. 1213m/sec

D. 1300.5 m/sec

E. 2088 m/sec

# MATH 1241
# COMMON FINAL EXAMINATION
# FREE RESPONSE SECTION
# FALL 2003

This exam is divided into three parts. These pages contain Part III which consists of 6 free response questions.

Please show all of your work on the problem sheet provided. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**

**At the end of the examination you MUST hand in this test booklet and all scratch paper.**

| PROBLEM | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| GRADE | | | | | | |

FREE RESPONSE SCORE: __________

Name: ______________________ Student ID No: ______________________

Instructor: ______________________ Section No: ______________________

## PART III (FREE RESPONSE SECTION)

1. For a certain function $f$, the following information is known:
   - i. $f$ is continuous, and has continuous first and second derivatives for all $x \neq 0$.
   - ii. $f'(x) > 0$ on $(-\infty, 0)$ and $(1, \infty)$; $f'(x) < 0$ on $(0, 1)$.
   - iii. $f''(x) > 0$ on $(-\infty, 0)$ and on $(0, \infty)$.
   - iv. $f(-1) = 4 = f(1)$.
   - v. $\lim_{x \to -\infty} f(x) = 0$; $\lim_{x \to \infty} f(x) = +\infty$.
   - vi. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = +\infty$.

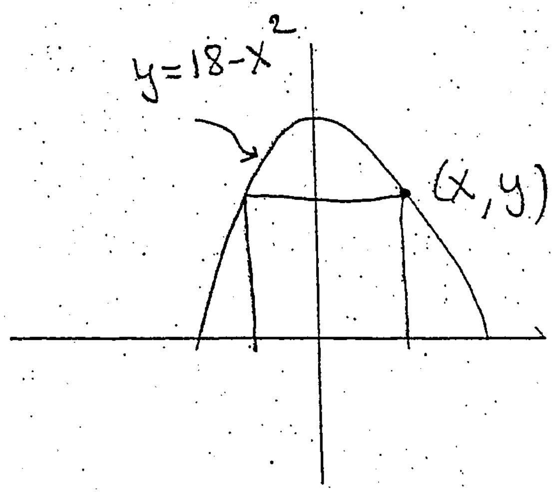
Use this information to answer the following questions:

(a) Determine where $f$ is increasing and where $f$ is decreasing, and find any local extreme points of $f$.

(b) Determine where the graph of $f$ is concave upward and where it is concave downward, and find any points of inflection.

(c) Sketch a curve which could be the graph of $f$.

2. Find the area of the largest rectangle which has one edge on the $x$-axis and two vertices on the parabola $y = 18 - x^2$, as in the illustration.



3. At the point on the curve $y = \cos x$ where $x = \pi/6$, the tangent line to the curve is drawn. At what exact point does this tangent line cross the $x$-axis?

4. (a) Find the limit

$$\lim_{x \to 0} \frac{\cos 3x - \cos 5x}{x^2}$$

(b) The limit

$$\lim_{x \to \pi/2} \frac{(\sin x) - 1}{x - \frac{\pi}{2}}$$

represents the derivative $g'(\pi/2)$ for a certain function $g$. Find the function $g$, and also the value of this limit.

5. A particle moves on a vertical line so that its coordinate at time $t$ is

$$y = t^3 - 12t + 3, \quad t \geq 0.$$

(a) Where is the particle at time $t = 0$?

(b) In which direction is the particle traveling at time $t = 1$?

(c) What are the highest and lowest points on the vertical line reached by the particle in the time interval $0 \leq t \leq 3.5$?

(d) Find the total distance traveled by the particle in the time interval $0 \leq t \leq 3.5$.

6. Consider all cubic polynomials of the form $f(x) = x^3 + cx^2 + x$.

(a) Find a value of $c$ for which $f$ is increasing for all $x$.

(b) Find a different value of $c$ for which $f$ has both a local maximum and a local minimum. For the value of $c$ you found, determine the $x$-coordinate of the local maximum and the $x$-coordinate of the local minimum.

(c) Show that every cubic polynomial $f$ of this form has exactly one inflection point.